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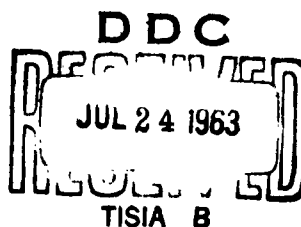
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**PATTERN PROPAGATION
ABOVE A CURVED EARTH
IN THE INTERFERENCE REGION**

C. F. Lowell



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by

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ABSTRACT

An analysis of the coverage pattern of transmitting equipment in the interference region requires a solution to the geometrical ray configuration which is valid within the effective line-of-sight of the transmitter. The limiting parameter on the accuracy required in the analysis is quite high for relatively short wave lengths of transmitted signal. The effects of the atmosphere and physical irregularities will be shown to modify an assumed geometric configuration. In addition, the results of the analysis will be shown solvable on a digital computer. The techniques of the solution will be indicated.

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CHAPTER I

INTRODUCTION

An analysis covering the area of pattern propagation will deal with the area of coverage of an airborne platform housing electronic equipment which transmits and receives electromagnetic waves of various types of modulation. The object of this paper will not be to analyze the airborne equipment itself, but rather to deal with the pattern of, or area of coverage of, the equipment in the earth's atmosphere, given the transmitting and receiving field strength functions of the platform. In other words, the properties of the airborne antenna will be an independent variable in determining the electromagnetic field strength at a given point in space relative to the airborne system. In general, the area to be covered will be an analysis of electromagnetic wave propagation from the time it leaves the airborne platform, until it reaches its target, and in the case of a radar system, returns to the platform again. Or for example, in the case of a radar platform located at x_0, y_0 above a reference to the earth, at what coordinates will a target be in the detection range of the radar?

This paper will consider two methods by which energy can reach a target from a transmitting platform and in general return again. One method is by direct line-of-sight from the platform to the target. The second is by considering a path which reflects from the surface of the earth. With both paths of transmission present at the same time, there will certainly be phase cancellation and reinforcement of the transmitted energy at the target. This fact alone means that if this effect is going to be taken into account, then a precise geometrical analysis is required if phase considerations are to be preserved and meaningful. However, the validity of the geometrical representation must be analyzed in order to adhere to the physical. For when the energy is of a wave length in the vicinity of one meter, then if the geometrical situation described above is not presented accurately by precise equations, the end result becomes entirely invalid. To

this end, much time and effort will be extended. Typical properties which must be taken into account are:

- (a) Divergence of energy from a spherical reflector.
- (b) Effect of the radius of the earth.
- (c) Atmospheric variations in the area involved.
- (d) The effect of polarization on reflection.

Other properties which will be analyzed are:

- (a) Reflector irregularities.
- (b) Ray tracing validity.

The author will perform an analysis of the above in relation to field strength versus spacial position from the platform by compiling equations which represent the factors involved. While this may appear as a mere task on the surface, if one looks further into the problem, he soon finds himself with a set of equations which become extremely cumbersome to solve without making simplifications which in turn render the results invalid insofar as phase considerations are concerned. Hence, the author's approach in deriving valid equations which are solvable, within the accuracy required, on a digital computer such as the IBM 7090 will be covered in the paper. The equations finally used will be discussed, as well as other possible approaches which were rejected for one reason or another. Of considerable practical interest will be the resulting general computer program to solve the composite equations.

CHAPTER II

FUNDAMENTAL CONCEPTS OF PROPAGATION

An analysis of any airborne equipment will usually involve the properties which will be discussed in the pages that follow. Consider first the transmission of electromagnetic energy in a theoretical free space. This would consist of transmission in a vacuum medium between objects remote from other influences that transmission is essentially unaffected.

A. Generalized Philosophy

In general, first to be considered is the isolated radiating antenna itself. Consider a polar axis of a spherical coordinate system such that its origin is the origin of the antenna. A point in space relative to the antenna is then characterized by specifying angles θ and ϕ with respect to the antenna. θ may be the zenith angle and ϕ the angle about the polar axis.

An antenna has radiating properties which are thus specified in terms of θ and ϕ . $F(\theta, \phi)$ may then be defined as the ratio of electric or magnetic field strength in the (θ, ϕ) direction to that which occurs in the direction of maximum field strength. Measurements are taken at a distance which is large compared to both the wave length and dimension of the antenna such that the near field of the antenna is neglected and the energy transmitted consists of constant phase wave fronts.

Considering radiation from the antenna, Poynting's vector ($\bar{P} = \bar{E} \times \bar{H}$), or the cross product of the electric and magnetic field, gives the magnitude of the energy flow per unit area in the direction of propagation when its magnitude is averaged over time. For the steady state sinusoidal consideration this average power can be expressed in complex notation as:

$$\bar{P}_{av} = \frac{1}{2} \text{Re}(\bar{E} \times \bar{H}^*) \quad (1)$$

The above notation assumes that the incremental section of the radiation can be considered as a plane wave front, which is true in the far field of the antenna. Utilizing MKS units, the instantaneous value of Poynting's vector is given as:

$$\bar{P} = \frac{E^2}{\eta_0} = \eta_0 H^2 \quad (2)$$

where E and H are the instantaneous values of the total electric and magnetic field strength and:

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 120\pi \text{ Ohms} \quad \text{impedance of free space} \quad (3)$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ henrys/meter} \quad (4)$$

$$\epsilon_0 = \frac{10^{-9}}{36\pi} \text{ farads/meter} \quad (5)$$

μ_0 is called the permeability and ϵ_0 the permittivity. The time average of Poynting's vector hence degenerates to:

$$\bar{P}_{av} = \frac{E^2}{2\eta_0} \quad (6)$$

as:

$$F(\theta, \phi) = \frac{E(\theta, \phi)}{E_0} \quad (7)$$

$$\bar{P}_{av}(\theta, \phi) = |F(\theta, \phi)|^2 \bar{P}_{av(0)} \quad (8)$$

where $\bar{P}_{av(0)}$ is the average power in the direction of maximum transmission.

At this point, it is useful to define an isotropic radiator as an antenna which radiates equally in all directions, hence for all values of θ and ϕ :

$$F(\theta, \phi) = 1.0 \quad (9)$$

For an isotropic radiator, Poynting's vector is given by the well-known relationship:

$$\bar{P}_{iso} = \frac{W_i}{4\pi R^2} \quad (10)$$

where W_1 is the average power radiated by the isotropic element in watts and R is the distance in question.

The power radiated by a directional antenna can be obtained from Equation (8). Denote this power as W_2 . The power in question is found as the surface integral of Equation (8) or more specifically as:

$$W_2 = \int_{4\pi} \bar{P}_{av}(\theta, \phi) R^2 d\sigma \quad (11)$$

where the surface in question is a sphere of radius R from the radiating element with surface area $R^2 d\sigma$ where σ is the solid angle in steradians as viewed from the antenna.

With the above in mind, the absolute antenna gain can be defined and obtained. The absolute antenna gain is defined as the ratio of the radiated power by the isotropic element required to produce a given value of Poynting's vector at a set distance to the power radiated by the directional element producing the same value of Poynting's vector at the same distance in the direction of maximum power radiation. Rewriting Equation (11) for a directional element yields:

$$W_2 = 4\pi R^2 \bar{P}_{av(0)} \int_{4\pi} |F(\theta, \phi)|^2 d\sigma \quad (12)$$

The absolute antenna gain is hence equal to:

$$G_t = \frac{W_1}{W_2} = \frac{\bar{P}_{iso} \int_{4\pi} d\sigma}{\bar{P}_{av(0)} \int_{4\pi} |F(\theta, \phi)|^2 d\sigma} \quad (13)$$

By employing the above definition, $\bar{P}_{(max)}$ of the directional antenna is equal to \bar{P}_{iso} and hence $W_1 = G_t W_2$. This leads to the conclusions that the maximum values of Poynting's vector for the directional case is equal to:

$$\bar{P}_{max} = \frac{W_1}{4\pi R^2} = \frac{W_2 G_t}{4\pi R^2} \quad (14)$$

As W_2 is the transmitted power, the substitution that $W_2 = W_t$ can be made. This leads to an expression for $\bar{P}(\theta, \phi)$ by utilizing Equation (8) and making the substitution that $\bar{P}_{(\max)} = \bar{P}_{av(0)}$:

$$\bar{P}_{av}(\theta, \phi) = \frac{W_t G_t}{4\pi R^2} |F(\theta, \phi)|^2 \quad (15)$$

Through the use of the cross product representation for Poynting's vector, Equation (2), values of the electric field strength as a function of position are found as:

$$E(\theta, \phi) = \sqrt{\bar{P}_{av} 120\pi} = \frac{\sqrt{60W_t G_t}}{R} F(\theta, \phi)$$

$$E(\theta, \phi) = E_0 F(\theta, \phi) \quad (16)$$

It should be noted that if the Hertzian dipole or the half-wave doublet are to be considered as the gain standards, then the values of total gain should be divided by $3/2$ and 1.64 respectively.

B. One-Way Transmission

For the preceding information to be of practical use, the transmitted energy must be received by a receiving element and receiver. If we consider a properly matched receiver and receiving antenna which are oriented in a direction for maximum reception, the received power W_r can be written as ^[1]:

$$W_r = \bar{P}_{av} A_e \quad (17)$$

where \bar{P}_{av} is the value of the received Poynting's vector and A_e the effective cross section of the receiving antenna. With the wave length of the transmitted energy expressed in meters (λ), the gain of the receiving element is given as:

$$G_r = \frac{4\pi A_e}{\lambda^2} \quad (18)$$

[1] S.A. Schelkunoff, Electromagnetic Waves, Van Nostrand, New York, 1943, Sec. 9.4.

If the receiving and transmitting antennas are of the same polarization and if the direction of the receiving array is specified relative to the incoming energy in terms of θ' and ϕ' , then the received power obtained by Equation (17) as a function of θ' and ϕ' must be multiplied by the square of the antenna pattern function, $F(\theta', \phi')$. This leads to the following:

$$W_r = \frac{\bar{P}_{av} G_r \lambda^2}{4\pi} |F(\theta', \phi')|^2 \quad (19)$$

or:

$$\begin{aligned} \frac{W_r}{W_t} &= \frac{G_t \bar{P}_G \lambda^2}{\bar{P} 4\pi R^2} |F(\theta', \phi')|^2 |F(\theta, \phi)|^2 \\ \frac{W_r}{W_t} &= \frac{G_t G_r \lambda^2}{(4\pi R)^2} |F(\theta, \phi) F(\theta', \phi')|^2 \end{aligned} \quad (20)$$

For the case stated previously where both antennas are aligned for maximum conditions, Equation (20) degenerates to:

$$\frac{W_r}{W_t} = \frac{G_t G_r \lambda^2}{(4\pi R)^2} \quad (21)$$

Solving for R in Equation (21) results in:

$$R = \sqrt{\frac{W_t}{W_r}} \sqrt{\frac{G_t G_r}{4\pi}} \lambda \quad (22)$$

This equation may be converted directly to the maximum free-space one-way transmission range or the target range at which a usable energy is received by the receiver by letting $G_r = G_{(max)}$, and utilizing the minimum useful receiver power. Equation (22) hence is transformed into:

$$R_0 = \sqrt{\frac{W_t}{W_{min}}} \sqrt{\frac{G_t G_{max}}{4\pi}} \lambda \quad (23)$$

where R_0 is the maximum theoretical range. This equation is used frequently as a standard of performance.

C. Radar Transmission

As this paper will be ultimately concerned with two-way transmission as well as one-way transmission, a match for Equation (23) will now be derived. Two-way transmission infers the concept of reflecting transmitted energy by a remote object from the transmitter and receiving a portion of this reflected energy at the transmitter. This type of transmission or radar transmission utilizes the properties of the radar target. It is first necessary in the development of radar transmission to specify the radar cross section or back-scattering cross sectional area. Let σ be the dimension of the back-scattering area. It will not be the object of this paper to specify either a mathematical or experimental determination of σ but only to assume that its fictional mathematical value is known. A definition of σ is the area intercepting that amount of power which, when isotropically scattered, produces an echo equal to that observed from the target [2].

Utilizing the previous definition, the value of the back-scattered Poynting's vector at a point from the target is related to σ as well as the incident Poynting's vector on the target in the following manner:

$$\bar{P}_{bs} = \frac{\bar{P}_i \sigma}{4\pi R^2} \quad (24)$$

or:

$$\sigma = \frac{\bar{P}_{bs}}{\bar{P}_i} 4\pi R^2 \quad (25)$$

where \bar{P}_i is the incident value and \bar{P}_{bs} the back-scattered value of the Poynting's vector respectively. R is the distance from the radar target. In a manner similar to that previously discussed, the received power by the radar antenna is:

$$W_r = |F_r(\theta', \phi)|^2 \frac{A_e \bar{P}_i \sigma}{4\pi R^2} \quad (26)$$

[2] D.E. Kerr, Propagation of Short Radio Waves, McGraw-Hill, New York, 1951, p. 33.

The value of the incident Poynting's vector is:

$$\bar{P}_i = \frac{W_t G_t}{4\pi R^2} |F(\theta, \phi)|^2$$

Assuming that the transmitting and receiving antennas for radar transmission are identical:

$$\frac{W_r}{W_t} = |F_r(\theta, \phi)|^2 |F(\theta, \phi)|^2 \frac{A_e \sigma G_t}{4\pi R^2 4\pi R^2} \quad (27)$$

where:

$$A_e = \lambda^2 \frac{G_r}{4\pi}$$

Substituting the value for A_e yields:

$$\frac{W_r}{W_t} = \frac{G^2 \lambda^2 \sigma}{(4\pi)^3 R^4} |F(\theta, \phi)|^4 \quad (28)$$

As before, a free-space radar range is defined as that range which produces a minimum radar signal which is useful. Solving Equation (28) for this range:

$$R^4 = \frac{W_t}{W_r} \frac{G^2 \lambda^2 \sigma}{(4\pi)^3} |F(\theta, \phi)|^4 \quad (29)$$

$$R_0 = \sqrt[4]{\frac{W_t \sigma}{W_r 4\pi} \sqrt{\frac{G\lambda}{4\pi}}} \quad (30)$$

Utilizing the preceding, a free-space coverage factor may be determined. Substituting Equation (30) into Equation (29) yields :

$$R = R_0 |F(\theta, \phi)| \quad (31)$$

This expression in general holds for both one-way and radar transmission. Utilizing the coverage factor, a free-space coverage diagram may be determined. This coverage

diagram encloses a volume inside which the electromagnetic field is larger than the smallest useful value.

D. The Propagation Factor

Up to this point, the free-space condition has been the major consideration. In a practical system, the coverage of transmitted energy may be largely affected by surrounding physical conditions. If reasonable parameters are used in the equations above and then the actual ranges determined from experimentation, there will be considerable discrepancy in the magnitudes of the quantities involved. The main reason for this discrepancy is that operation in the atmosphere close to the earth does not compare with the conditions assumed for free-space transmission. In an effort to analyze the true situation, the free-space relationships derived will not be disregarded but instead they will be used as the building block in describing the new or actual situation.

Consider first a factor which shall be called the pattern-propagation factor. This factor is defined as the ratio of the amplitude of the electric field at a given point under certain conditions to the amplitude of the electric field under the free-space ideal conditions where the beam of the transmitter is directed to the point under consideration. In equation form, the above is restated as:

$$F = \left| \frac{E}{E_0} \right| \quad (32)$$

where F is the propagation factor, E the electric field at the point in question, and E_0 the reference free-space electric field having the same polarization as E . F can be seen to be entirely independent of internal properties of the transmitter and dependent only upon the properties of the path of transmission and the directive properties of the antenna.

In the presence of an earth, F is the effect of the following: (1) the diffraction phenomenon or where the earth's shadow gives rise to diffraction of electromagnetic energy; (2) the refraction phenomenon or the effects of an inhomogeneous atmosphere about the earth; and (3) the interference phenomenon which is the effect of the earth reflecting and scattering radiation which produces an interference pattern. The fourth

factor which F takes into account is the directional characteristics of the antenna; or for a free-space consideration F would be:

$$F = |F(\theta, \phi)| \quad (33)$$

Hence, the one-way transmission equation may now be written in terms of the above effects as:

$$\frac{W_r}{W_t} = \frac{G_t G_r \lambda^2}{(4\pi R)^2} F^2 |F_r(\theta', \phi')|^2 \quad (34)$$

Notable is the fact that F^2 is the ratio of the magnitudes of the Poynting's vector at the point in question to the magnitude of the Poynting's vector whose maximum free-space value corresponds to the same absolute distance as the point in question.

Not quite so easy to verify is the application of F into the radar range equation in order to make calculations that are valid for regions other than free space. Using the theory applied by H. A. Lorentz^[3] in the development of the reciprocity theorem it can be shown that the isotropic source equivalent to the radar target is $(W_t G_t / 4\pi R^2) F^2$. Also, using this theorem it can be proven that the propagation factor squared is the quantity that converts the free-space transmitted energy into the energy that would come from the target, if the target-reflected energy were incident on the maximum antenna pattern of the receiver. Hence, the radar transmission equation can be rewritten as:

$$\frac{W_r}{W_t} = \frac{G^2 \lambda^2 \sigma}{(4\pi)^3 R^4} F^4 \quad (35)$$

In Equations (34) and (35) are the theoretical capabilities for determining field strength calculations in the presence of the earth and its atmosphere. These equations are subject only to the conditions that the transmitted energy be incident upon either antenna or target as a plane electromagnetic wave.

While Equations (34) and (35) look relatively easy to solve in nature, it must not be forgotten that the secret to their practicality is a correct formulation of F . In general, it is to this desired end that a large portion of this paper will be concerned.

[3] Ibid., pp. 693-699.

propagation it is immediately seen that the energy arriving at point (2) in space is made up of a direct energy wave front and a reflected energy wave front which, due to their difference in distances traveled, are not of the same phase. Or, in terms of the E and H vector representation, the vectors representing the direct ray have not made as many complete revolutions as have the vectors representing the scattered ray. For a more technical discussion of the theory represented above, the reader is referred to any standard text on field and wave theory. ^[4]

Hence, in view of the preceding, it can be seen that the path length difference between the direct transmitted energy and the scattered transmitted energy received by the receiver in the case of one-way transmission, or the equal resulting effect in the case of a radar type transmission, must be considered as a component part of F. Or more explicitly, in order to show the qualitative properties of the pattern-propagation factor F in the interference region, consider again the situation shown as Figure 1. Here will be seen the transmitting antenna having a vertical plane pattern of $F(\phi)$ located at a height Z_1 above a spherical earth. The angles ϕ_1 and ϕ_2 are therefore needed to determine the antenna patterns $f(\phi_1)$ and $f(\phi_2)$ in the direction of the direct and scattered rays respectively. Note that it has been assumed that the energy wave front can be represented as a pencil-like ray or composed of vectors normal to the wave fronts and in the direction of propagation. The validity of the assumption will be discussed at a later time.

Consider that in the direction of maximum propagation of the antenna the electromagnetic field due to the direct ray is E_d . Then it follows that from the definition of $f(\phi) = F(\theta, \phi)$ that the value of the direct ray at an angle ϕ_1 (where the horizontal dependence has been omitted) is:

$$E_d = E_0 f(\phi_1) \quad (36)$$

where ϕ_1 and ϕ_2 are relative to the normal of Z_1 . The distance traveled by the direct ray to the receiving antenna is R_2 .

[4] S. Ramo and J. R. Whinnery, Fields and Waves in Modern Radio, John Wiley and Sons, New York, 1953, pp. 270-314.

In a similar manner, the value of the reflected wave as it leaves the transmitting antenna is:

$$E_r = E_0 f(\phi_2) \quad (37)$$

The path length of the reflected or scattered ray is $R_1 + R_3$. The direct and scattered rays thus travel a path length difference of:

$$\Delta R = R_1 + R_3 - R_2 \quad (38)$$

where $\Delta R > 0$ for all locations of the receiver in the interference region. Because of this path length difference, the scattered ray is retarded in phase by a value $(\Delta R)A$ where A is the quantity which converts the path length difference into its equivalent difference in terms of the wave length of the propagated energy.

The reflected wave also undergoes a phase retardation because it is reflecting from the surface of the earth. Not only does the reflected wave suffer retardation but its magnitude is affected by the reflection. At this point, define Γ as the ratio of the incident field, electric or magnetic, to the reflected field. This is a complex quantity expressing both the phase and magnitude changes suffered by a wave of energy reflecting from the surface of the earth. Or, in terms of symbolic complex notation:

$$\Gamma = \rho e^{-i\phi}$$

where ρ expresses the fraction of the incident field that is reflected and ϕ is the angle by which the incident energy is retarded.

If the difference in path lengths is expressed as the equivalent phase retardation, $A(\Delta R)$, and ϕ expressed in a similar manner, then the total phase retardation of the reflected complex field relative to the direct complex field is given as:

$$\Delta \psi = A(\Delta R) + \phi \quad (39)$$

where $\Delta \psi$ expresses the total phase retardation.

If we take into account the divergence of energy due to its incidence on a spherical reflector and this quantity is denoted as D , then the total electric field strength at the

receiving point located at Z_2 above the earth can be expressed as:

$$\bar{E}_t = \bar{E}_r + \bar{E}_d \quad (40)$$

Or in terms of our previous notation:

$$E_t = E_0 \left[f(\phi_1) + \Gamma D e^{-iA(\Delta R)} f(\phi_2) \right] \quad (41)$$

Hence, within its limitations and assumptions, the ratio of E to E_0 gives the interference pattern as formed in space, or the pattern propagation factor.

By limitations and assumptions is meant a wide scattering of theoretical and physical effects which for propagation in an atmosphere have been neglected in the previous analysis. Most of these effects will be discussed in Chapter V of this paper. However, there are two factors which will be mentioned at this point as no further consideration will be made of them. First, at the receiving point, the energy wave fronts will not be arriving at the same directions as has been assumed in the preceding analysis. The direct and scattered wave fronts will in actuality be traveling in slightly different directions. However, as found from experimentation, this is important only when the energy wave front is vertically polarized and the reflection and incident angles with respect to the earth differ to a large extent. In the paper these angles will be made or assumed equal according to the well known law:

$$\frac{\nu_1}{\sin \alpha_1} = \frac{\nu_2}{\sin \alpha_2}$$

where ν_1 and ν_2 are the velocities of propagation in the incident and reflected mediums and α_1 and α_2 the incident and reflected angles. As the incident and reflected waves are traveling in the same medium, $\nu_1 = \nu_2$ and hence:

$$\alpha_1 = \alpha_2 \quad (42)$$

The second factor which will be neglected is the small spatial attenuation difference encountered by the direct and reflected energy. In the large majority of practical cases, this effect has been found to be negligible^[5].

[5] L. N. Ridenour, Radar System Engineering, McGraw-Hill, New York, 1947, pp. 58-62.

CHAPTER III

THE FORMULATION OF THE DIFFERENCE BETWEEN
THE PHASE OF DIRECT AND SCATTERED ENERGY DUE
TO PATH LENGTH DIFFERENCE — ΔR

Referring to Equation (41), it is seen that one factor which must be found is ΔR . Most of the remainder of this paper will be concerned with finding these variable parameters as a function of Z_2 , Z_1 and R_2 . The reason for this is that the desired end result is a coverage diagram represented in terms of Z_2 and R_2 , with Z_1 as a fixed parameter. There has been considerable effort devoted by various concerns and individuals in the determining of ΔR as a function of the previously mentioned variables. Some of these methods will now be presented as a background for the final method used in this paper.

A. The Exact Equations

The geometrical situation described in Figure 1 on page 12 can be represented in exact forms by the following set of equations:

$$\alpha_1 = \alpha_2 \quad (43)$$

$$E_i = 1 + \left(\frac{Z_i}{A_e} \right) \quad i = 1, 2 \quad (\text{an intervening parameter}) \quad (44)$$

$$R_2 = A_e \sqrt{E_1^2 + E_2^2 - 2E_1E_2 \cos \theta_1} \quad (45)$$

$$\begin{aligned} & 4E_1E_2 \cos^4 \theta_2 - (4E_1E_2^2 + 4E_1^2E_2 \cos \theta_1) \cos^3 \theta_2 + (E_1^2 + E_2^2 + 2E_1E_2 \cos \theta_1 - 4E_1^2E_2^2) \cos^2 \theta_2 \\ & + (4E_1E_2^2 \cos^2 \theta_1 + 2E_1E_2^2 \sin^2 \theta_1 + 4E_1^2E_2 \cos \theta_1) \cos \theta_2 \\ & + (E_1^2E_2^2 \sin^2 \theta_1 - E_2^2 \cos^2 \theta_1 - E_1^2 - 2E_1E_2 \cos \theta_1) = 0 \end{aligned} \quad (46)$$

$$Z_1' = (A_e + Z_1) \cos \theta_2 - A_e \quad (47)$$

$$Z'_2 = (A_e + Z_2) \cos \theta - A_e \quad (48)$$

$$d = d_1 + d_2 = \sqrt{R_2^2 - (Z'_2 - Z'_1)^2} \quad \text{See Equation (58)} \quad (49)$$

$$\psi_2 = \tan^{-1} \left(\frac{Z'_1}{d_1} \right) \quad (50)$$

$$\psi_2 = \tan^{-1} \frac{(Z'_1 + Z'_2)}{d}$$

$$A_1 = \sqrt{A_e^2 \sin^2 \psi_2 + 2A_e Z_1 + Z_1^2} \quad (51)$$

$$A_2 = \sqrt{A_e^2 \sin^2 \psi_2 + 2A_e Z_2 + Z_2^2} \quad (52)$$

$$R_1 = A_1 - A_e \sin \psi_2 \quad (53)$$

$$R_3 = A_2 - A_e \sin \psi_2 \quad (54)$$

$$\Delta R = R_1 + R_3 - R_2 \quad (55)$$

$$\Delta R = A_1 + A_2 - 2A_e \sin \psi_2 \quad (56)$$

$$L = \sqrt{2A_e Z_1 + Z_1^2} + \sqrt{2A_e Z_2 + Z_2^2} \quad (57)$$

$$d_1 = (A_e + Z_1) \sin \theta = r \frac{Z'_1}{Z'_1 + Z'_2} \quad (58)$$

$$d_2 = (A_e + Z_2) \sin \theta = r \frac{Z'_2}{Z'_1 + Z'_2}$$

These equations are referred to as exact in that they represent the geometrical situation represented by Figure 1. Theoretically they may not be exact because they deviate from what is physically happening or because they cannot be solved to the required precision. The derivations of all formulae are straightforward from the geometrical viewpoint and may be checked by the reader if desired.

The use and practical application of these equations may be questioned. For an explicit solution of ΔR as a function of Z_1 , Z_2 and R_2 one would first proceed to solve the quadratic in $\cos \theta_2$ from Equation (46). This would give the $\cos \theta_2$ as a function of Z_1 , Z_2 , and $\cos \theta_1$. Equation (45) would then be solved for $\cos \theta_1$. Making appropriate substitutions, ΔR as a function Z_1 , Z_2 and R_2 can then be determined. Equation (57) gives a measure of the section in which the equations are valid, or Z_2 is in the interference region. This means that if R_2 is greater than L , then Z_2 has a minimum value greater than zero. While theoretically the preceding is possible, at its best it is very complicated. Also, in order to maintain the values R_1 and R_3 to the necessary degree of precision in terms of the wave length of propagation involved, the number of accurate digits one is required to maintain can be seen to be unwieldy.

Variations in the exact geometrical representation or equations have been derived by various writers. Most of these involve implicit relations which indeed are easier to work with than those described previously but their use is limited in the plotting of a coverage diagram in terms of the parameters previously stated. The University of Michigan^[6] and the Fairchild Engine and Airplane Corporation^[7] have both contributed concepts not generally used in the preceding equations. Equation (12) in the Fairchild report is questionable, should reference be made to this paper. The author has not, however, proved it incorrect even though dimensional analysis indicates an error.

[6] T. B. Curtz, M. L. Barasch, et al, "Analysis of Padar and Its Modifications — Final Report, " No. 2476-1-F, April, 1956.

[7] "Padar Investigation — Final Report, " No. 64R-10, June, 1956.

B. The Cubic in r_1

Admittedly, there are simplifications that can be made in the so-called "exact" equations. One of these may be termed as a cubic in r_1 . Consider the following approximation:

$$Z_1 \text{ and } Z_2 \ll A_e \quad (59)$$

$$r_1 \text{ and } r_2 \ll A_e \quad (60)$$

Therefore:

$$Z'_1 \cong Z_1 - \frac{r_1^2}{2A_e} \quad (61)$$

$$Z'_2 \cong Z_2 - \frac{r_2^2}{2A_e} \quad (62)$$

$$d_1 \cong r_1, \quad d_2 \cong r_2, \quad d \cong r \quad (63)$$

$$\psi_2 \cong \tan^{-1} \left\{ \left[\frac{(Z_1 + Z_2)}{r} \right] \left[1 - \frac{r_1^2 + r_2^2}{2A_e(Z_1 + Z_2)} \right] \right\} \quad (64)$$

$$r_2 \cong \frac{r_1^2 - 2A_e Z_1}{2r_1} + \sqrt{\left(\frac{r_1^2 - 2A_e Z_1}{2r_1} \right)^2 + 2A_e Z_2} \quad (65)$$

$$2r_1^3 - 3rr_1^2 + (r - 2A_e Z_1 - 2A_e Z_2)r_1 + 2A_e Z_1 r \cong 0 \quad (66)$$

Equations (61) and (62) follow directly from Equations (47) and (48) by letting

$\cos \theta \cong 1 - \frac{\theta^2}{2}$. Likewise, Equation (63) results from Equations (58) and (49) by letting $\sin \theta \cong 0$. Equation (64) results from Equation (50) utilizing Equations (61), (62), and (63). Equations (65) and (66) follow from (64) assuming that (64) is valid. Equation (66) has the solution:

$$r_1 = \frac{r}{2} - P \cos \left\{ \frac{(\phi + \pi)}{3} \right\} \quad (67)$$

where:

$$P = \frac{2\sqrt{A_e(Z_1 + Z_2) + \left(\frac{r}{2}\right)^2}}{\sqrt{3}} \quad (68)$$

$$\phi = \cos^{-1} \left\{ \frac{2A_e(Z_2 - Z_1)r}{P^3} \right\} \quad (69)$$

$Z_1 \geq Z_2$

For $Z_1 < Z_2$ a similar equation can be found in terms of r_2 . This solution is essentially that given by Kerr ^[8] on page 113 except for the sign change in:

$$r_1 = \frac{r}{2} + P \cos \left\{ \frac{(\phi + \pi)}{3} \right\} \quad (70)$$

This can be shown to be incorrect if we observe that as $Z_1 \geq Z_2$, the following inequalities must hold:

$$r_1 - \frac{r}{2} \geq 0. \quad (71)$$

$$\cos \phi \leq 0. \quad (72)$$

However, as:

$$270^\circ > \phi > 90^\circ \quad (73)$$

$$150^\circ > \frac{\phi + \pi}{3} > 90^\circ$$

$$\cos \left\{ \frac{(\phi + \pi)}{3} \right\} \leq 0 \quad (74)$$

[8] Kerr, op. cit., p. 113.

Hence for (71) to hold true, Equation (70) must be incorrect and the form must be that of Equation (67). The law of cosines may be invoked to find R_1 and R_3 .

$$R_1^2 = A_e^2 + (A_e + Z_1)^2 - 2A_e(A_e + Z_1) \cos \theta_1 \quad (75)$$

$$\theta_1 = \frac{r - r_2}{A_e} = \frac{r_1}{A_e} \quad (76)$$

$$R_3^2 = A_e^2 + (A_e + Z_2)^2 - 2A_e(A_e + Z_2) \cos \theta \quad (77)$$

$$\theta = \frac{r - r_1}{A_e} \quad (78)$$

Therefore:

$$\begin{aligned} \Delta R = & -R_2 + \sqrt{A_e^2 + (A_e + Z_1)^2 - 2A_e(A_e + Z_1) \cos \frac{r_1}{A_e}} \\ & + \sqrt{A_e^2 + (A_e + Z_2)^2 - 2A_e(A_e + Z_2) \cos \frac{r - r_1}{A_e}} \end{aligned} \quad (79)$$

It should be noted that the cubic in r_1 does not represent a full third-order approximation to r_1 . To obtain this, the third-order term in the expansion of $\sin \theta$ or $\sin \theta_1$ must be used in the approximation of d_1 and d_2 . This will not only change the coefficient of r_1 but will also change the remaining coefficients. However, as Z_1/A_e and Z_2/A_e are small, these changes are unimportant. Theoretically, these equations break down to the largest extent at the extreme conditions or where Z_2 is too large or where R_2 is at either extreme.

Considerable work has been done on obtaining the solution to the cubic in r_1 . Transformations have been found which normalize the variables in such a way that the

computing required for a solution has been reduced considerably. The application of this transformation is recommended and may be found in Fishback^[9] and Burrows and Atwood^[10].

C. The Flat Earth Approximation

A third method of attack in determining ΔR in the interference region can be found by assuming a flat earth or an earth whose radius is infinite. Under this assumption:

$$Z'_1 = Z_1 \quad Z'_2 = Z_2 \quad (80)$$

$$\psi_2 = \tan^{-1} \left(\frac{Z_1}{r_1} \right) \text{ or } \tan^{-1} \left(\frac{Z_2}{r_2} \right) \quad (81)$$

$$\psi_2 = \tan^{-1} \frac{Z_1 + Z_2}{r} \quad (82)$$

If Z_1/r_1 and Z_2/r_2 are assumed small, then:

$$R_1 \cong r_1 + \frac{Z_1^2}{2r_1} \cong r_1 \quad (83)$$

$$R_3 \cong r_2 + \frac{Z_2^2}{2r_2} \cong r_2 \quad (84)$$

$$r_1 \cong \frac{Z_1 r}{Z_1 + Z_2} \quad (85)$$

[9] W. T. Fishback, "Simplified Methods of Field Intensity Calculations in the Interference Region," Report No. 461, 1943.

[10] C. R. Burrows and S. S. Atwood, Radio Wave Propagation, Academic Press, New York, 1949.

$$r_2 \cong \frac{Z_2 r_1}{Z_1} \quad (86)$$

$$\Delta R \cong \frac{2Z_1 Z_2}{r} \quad (87)$$

Equation (87) can be seen from the fact that pure geometrical deduction indicates:

$$R_2 = \sqrt{r^2 + (Z_2 - Z_1)^2}$$

$$(R_1 + R_3) = \sqrt{r^2 + (Z_2 + Z_1)^2}$$

Expanding R_2 and $(R_1 + R_3)$ by the binomial theorem and subtracting yields:

$$\Delta R = \frac{2Z_1 Z_2}{r} \left(1 - \frac{Z_1^2 + Z_2^2}{2r^2} + \frac{3Z_1^4 + 10Z_1^2 Z_2^2 + 3Z_2^4}{8r^4} - \dots \right)$$

or:

$$\Delta R \cong \frac{2Z_1 Z_2}{r} \quad (\text{Assuming Remaining Terms Small})$$

Needless to say, these equations break down everywhere that the previous equations did plus their additional inherent loss of accuracy brought about by the extended assumptions and simplifications. Notable, however, is that the cubic in r_1 is now replaced by (85) and (86). Of particular interest is the fact that these equations break down very rapidly in the region where R_2 approaches or is greater than L and Z_2 is small.

Reference will be made in a later section of the paper as to the quantitative magnitudes of the errors involved in these and the previous assumptions.

D. The Author's Analysis — The Exact Transcendental Equations

Three methods of determining ΔR have so far been presented. Each succeeding approach is simpler in terms of the computations required to obtain a result. However, the simplifications are obtained at an increasing loss of accuracy. It is the author's intent in this section to derive equations which will yield the desired ΔR to a high degree of accuracy comparable to that obtained theoretically by the fourth order "exact" equations but at the same time not to have the equations too unwieldy.

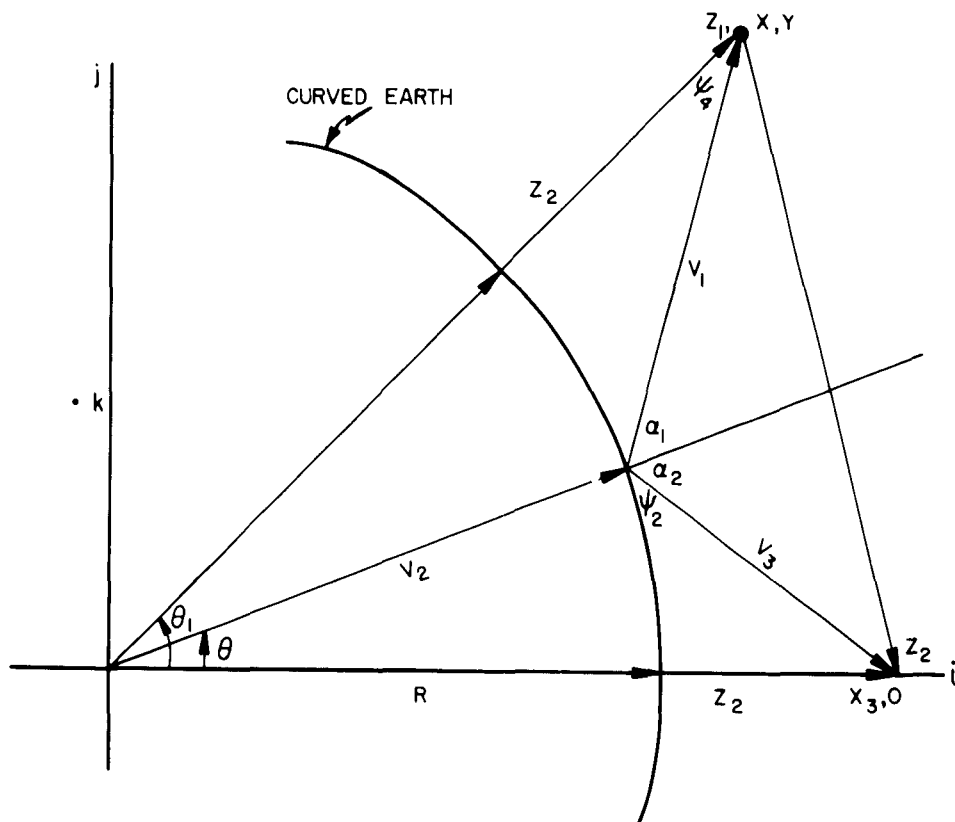


Figure 2. Vector Representation of Interference Region

Consider Figure 2 which is essentially the same as Figure 1 drawn in vector notation. The vectors \bar{V}_1 , \bar{V}_2 , and \bar{V}_3 may be expressed in the following manner:

$$\bar{V}_1 = (x - R \cos \theta)i + (y - R \sin \theta)j \quad (88)$$

$$\bar{V}_2 = (R \cos \theta)i + (R \sin \theta)j \quad (89)$$

$$\bar{V}_3 = (x_3 - R \cos \theta)i + (-R \sin \theta)j \quad (90)$$

Forming the following scalar or dot products results in:

$$\begin{aligned} \bar{V}_1 \cdot \bar{V}_2 &= xR \cos \theta - R^2 \cos^2 \theta + Ry \sin \theta - R^2 \sin^2 \theta \\ &= xR \cos \theta + yR \sin \theta - R^2 (\sin^2 \theta + \cos^2 \theta) \\ &= xR \cos \theta + yR \sin \theta - R^2 \end{aligned} \quad (91)$$

$$\begin{aligned} \bar{V}_3 \cdot \bar{V}_2 &= x_3 R \cos \theta - R^2 \cos^2 \theta - R^2 \sin^2 \theta \\ &= x_3 R \cos \theta - R^2 (\sin^2 \theta + \cos^2 \theta) \\ &= x_3 R \cos \theta - R^2 \end{aligned} \quad (92)$$

However, by the definition of the scalar or dot product of two vectors:

$$\bar{V}_1 \cdot \bar{V}_2 = |\bar{V}_1| |\bar{V}_2| \cos \alpha_1 \quad (93)$$

$$\bar{V}_3 \cdot \bar{V}_2 = |\bar{V}_3| |\bar{V}_2| \cos \alpha_2 \quad (94)$$

$$\therefore |\bar{V}_1| \cos \alpha_1 = \frac{xR \cos \theta + yR \sin \theta - R^2}{|\bar{V}_2|} \quad (95)$$

But :

$$|V_2| = R \quad (96)$$

$$\therefore |V_1| \cos \alpha_1 = x \cos \theta + y \sin \theta - R \quad (97)$$

$$|V_3| \cos \alpha_2 = x_3 \cos \theta - R \quad (98)$$

$$\cos \alpha_1 = \frac{x \cos \theta + y \sin \theta - R}{|V_1|} \quad (99)$$

$$\cos \alpha_2 = \frac{x_3 \cos \theta - R}{|V_3|} \quad (100)$$

or:

$$\cos \alpha_1 = \frac{x \cos \theta + y \sin \theta - R}{\sqrt{(x - R \cos \theta)^2 + (y - R \sin \theta)^2}} \quad (101)$$

$$\cos \alpha_2 = \frac{x_3 \cos \theta - R}{\sqrt{(x_3 - R \cos \theta)^2 + (R \sin \theta)^2}} \quad (102)$$

Using the following three identities:

$$(1) \quad x = (R + Z_1) \cos \theta_1,$$

$$(2) \quad y = (R + Z_1) \sin \theta_1,$$

$$(3) \quad x_3 = R + Z_2$$

It is easily proven that:

$$\begin{aligned} \cos^2 \alpha_1 &= \frac{(R + Z_1)K - 2R + R^2/(R + Z_1)K}{(R + Z_1)1/K - 2R + R^2/(R + Z_1)K} \\ &= \frac{(1 + Z_1/R)K + R/(R + Z_1)K - 2}{(1 + Z_1/R)1/K + R/(R + Z_1)K - 2} \end{aligned} \quad (103)$$

where $K = \cos(\theta - \theta_1)$

$$\begin{aligned} \cos^2 \alpha_2 &= \frac{(R + Z_2) \cos \theta - 2R + R^2 / (R + Z_2) \cos \theta}{(R + Z_2) / \cos \theta - 2R + R^2 / (R + Z_2) \cos \theta} \\ &= \frac{(1 + Z_2/R) \cos \theta + R / (R + Z_2) \cos \theta - 2}{(1 + Z_2/R) 1 / \cos \theta + R / (R + Z_2) \cos \theta - 2} \end{aligned} \quad (104)$$

The previous equations for α_1 and α_2 are at their best not as simple as might be desired. Assuming that θ_1 is known, which will be proven at a later time, and knowing from the previous discussion that $\cos^2 \alpha_2 = \cos^2 \alpha_1$, the previous two equations can theoretically be solved for θ , hence α_1 and α_2 . However, the solution for θ is an implicit relationship. It would, hence, be desirable to at least attempt to simplify Equations (103) and (104).

With this goal in mind, consider the following triple scalar product:

$$\bar{V}_3 \times \bar{V}_2 \cdot k = |V_2| |V_3| \sin \alpha_2 \quad (105)$$

$$\bar{V}_2 \times \bar{V}_1 \cdot k = |V_2| |V_1| \sin \alpha_1 \quad (106)$$

Expanding (105):

$$\begin{aligned} \bar{V}_3 \times \bar{V}_2 \cdot k &= \begin{vmatrix} x_3 - R \cos \theta & -R \sin \theta & 0 \\ R \cos \theta & R \sin \theta & 0 \\ 0 & 0 & 1 \end{vmatrix} \\ &= (x_3 - R \cos \theta) (R \sin \theta) + R \cos \theta (R \sin \theta) \\ &= x_3 R \sin \theta \end{aligned} \quad (107)$$

where k represents the unit vector in the third dimension.

$$\begin{aligned}
\bar{V}_2 \times \bar{V}_1 \cdot k &= \begin{vmatrix} R \cos \theta & R \sin \theta & 0 \\ x - R \cos \theta & y - R \sin \theta & 0 \\ 0 & 0 & 1 \end{vmatrix} \\
&= R \cos \theta (y - R \sin \theta) - (x - R \cos \theta) (R \sin \theta) \\
&= Ry \cos \theta - xR \sin \theta
\end{aligned}
\tag{108}$$

Utilizing the definition of the triple scalar product:

$$\begin{aligned}
\bar{V}_2 \times \bar{V}_1 \cdot k &= |V_1| |V_2| (1) (\sin \alpha_1) (\cos 0^\circ) \\
&= |V_1| |V_2| \sin \alpha_1
\end{aligned}
\tag{109}$$

$$\begin{aligned}
\bar{V}_3 \times \bar{V}_2 \cdot k &= |V_3| |V_2| (1) (-\sin \alpha_2) (\cos 0^\circ) \\
&= |V_3| |V_2| (-\sin \alpha_2)
\end{aligned}
\tag{110}$$

Note :

$$\bar{V}_1 \times \bar{V}_2 \neq \bar{V}_2 \times \bar{V}_1$$

But :

$$\bar{V}_1 \times \bar{V}_2 = -(\bar{V}_2 \times \bar{V}_1)$$

Rewriting (93) and (94):

$$|V_3| |V_2| = \frac{\bar{V}_3 \cdot \bar{V}_2}{\cos \alpha_2}$$

$$|V_1| |V_2| = \frac{\bar{V}_1 \cdot \bar{V}_2}{\cos \alpha_1}$$

$$\therefore \bar{V}_3 \times \bar{V}_2 \cdot k = \bar{V}_3 \cdot \bar{V}_2 \left(\frac{-\sin \alpha_2}{\cos \alpha_2} \right) \quad (111)$$

$$\bar{V}_2 \times \bar{V}_1 \cdot k = \bar{V}_1 \cdot \bar{V}_2 \left(\frac{\sin \alpha_1}{\cos \alpha_1} \right) \quad (112)$$

Therefore :

$$\tan \alpha_1 = \frac{\bar{V}_2 \times \bar{V}_1 \cdot k}{\bar{V}_1 \cdot \bar{V}_2} \quad (113)$$

$$\tan \alpha_2 = \frac{-\bar{V}_3 \times \bar{V}_2 \cdot k}{\bar{V}_3 \cdot \bar{V}_2} \quad (114)$$

Utilizing Equations (91), (92), (107), and (108):

$$\begin{aligned} \tan \alpha_1 &= \frac{Ry \cos \theta - xR \sin \theta}{xR \cos \theta - yR \sin \theta - R^2} \\ &= \frac{y \cos \theta - x \sin \theta}{x \cos \theta + y \sin \theta - R} \end{aligned} \quad (115)$$

$$\begin{aligned} \tan \alpha_2 &= \frac{-x_3 R \sin \theta}{x_3 R \cos \theta - R^2} \\ &= \frac{-x_3 \sin \theta}{x_3 \cos \theta - R} \end{aligned} \quad (116)$$

As:

$$\begin{aligned} x &= (R + Z_1) \cos \theta_1, \\ y &= (R + Z_1) \sin \theta_1, \\ x_3 &= R + Z_2 \end{aligned}$$

and using the trigonometric identities for the sine and cosine of the sum of two angles, the following is obtained:

$$\tan \alpha_1 = \frac{\sin(\theta_1 - \theta)}{\cos(\theta_1 - \theta) - R/(R + Z_1)} \quad (117)$$

$$\tan \alpha_2 = \frac{\sin \theta}{-\cos \theta + R/(R + Z_2)} \quad (118)$$

As transcendental equations, these equations are relatively simple in view of the previous results. From the previous derivations, the absolute value of \bar{V}_1 (or R_1) and the absolute value of \bar{V}_3 (or R_3) can be determined. From Equations (97) and (98):

$$|V_1| = \frac{x \cos \theta + y \sin \theta - R}{\cos \alpha_1} \quad (119)$$

$$|V_3| = \frac{x_3 \cos \theta - R}{\cos \alpha_2} \quad (120)$$

Substituting in the value for x, y and x_3 :

$$|V_1| = \frac{(R + Z_1) \cos \theta_1 \cos \theta + (R + Z_1) \sin \theta_1 \sin \theta - R}{\cos \alpha_1}$$

$$|V_3| = \frac{(R + Z_2) \cos \theta - R}{\cos \alpha_2}$$

Let

$$Z = \cos(\theta_1 - \theta)$$

$$R = A_e$$

Then:

$$|V_1| = R_1 = \left\{ \frac{[(Z - 1) + (Z_1 \times Z) / A_e]}{\cos \alpha_1} \right\} A_e \quad (121)$$

$$|V_3| = R_3 = \left\{ \frac{[(\cos \theta - 1) + (Z_2 \cos \theta) / A_e]}{\cos \alpha_2} \right\} A_e \quad (122)$$

To complete the analysis of ΔR , it is now necessary to derive an expression for θ_1 . This is a relatively simple procedure as it results directly from the solution of an oblique triangle. Given the following triangle denoted as Figure 3:

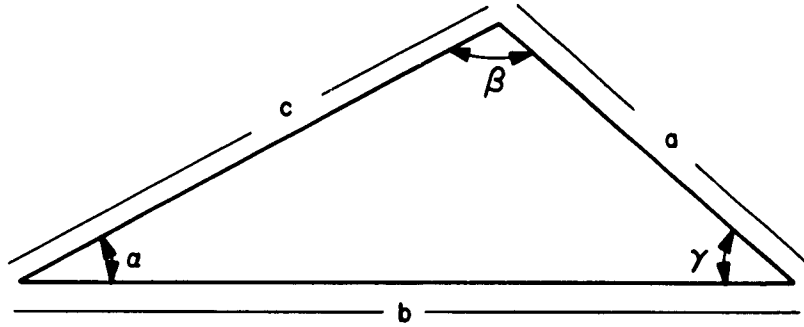


Figure 3. Triangle Solution of θ_1 in General Terms

the angle included between two known sides $b + c$ may be found as:

$$\alpha = 2 \tan^{-1} \left(\frac{r}{s - a} \right)$$

where:

$$s = \frac{1}{2} (a + b + c)$$

$$r = \sqrt{\frac{(s - a)(s - b)(s - c)}{s}}$$

For the situation pictured in Figure 1 :

$$s = \frac{2A_e - R_2 + Z_1 + Z_2}{2} \quad (123)$$

$$r = \sqrt{\frac{(s - R_2)(s - A_e - Z_1)(s - A_e - Z_2)}{s}} \quad (124)$$

$$\therefore \theta_1 = 2 \tan^{-1} \sqrt{\frac{(s - A_e - Z_1)(s - A_e - Z_2)}{(s - R_2)s}} \quad (125)$$

The logical procedure to follow when solving for ΔR from these equations is to first solve Equations (123), (124) and (125) knowing A_e , R_2 , Z_1 and Z_2 . Having once solved for θ_1 , then Equations (117) and (118) may be solved for α_1 and α_2 . Equation (121) and Equation (122) may then be solved directly for R_1 and R_3 . ΔR is then found as:

$$\Delta R = R_1 + R_3 - R_2$$

The only difficulty of any consequence when solving these equations when compared with the previous exact formulations lies in the solution of Equations (117) and (118). It has been found by the author that these equations are solved rapidly by using the following technique. First, calculate a value of ψ_2 by using the flat earth approximation:

$$\tan \psi_2 = \left(Z_1 + \frac{Z_2}{R_2} \right) \quad (126)$$

by the approximation $r \cong R_2$:

$$r_2 = \frac{Z_2}{\tan \psi_2} \quad (127)$$

$$\theta = \frac{r_2}{A_e} \quad (128)$$

Having thus found an approximate value for θ , calculate the tangents of α_1 , and α_2 by Equations (117) and (118). Then find the difference between the tangents of α_1 and α_2 and call this D. Assuming that $D \neq 0$, then the following may be stated, assuming a flat earth:

$$D = \frac{r_1}{Z_1} - \frac{r_2}{Z_2} = \frac{r}{Z_1} - r_2 \left(\frac{1}{Z_1} + \frac{1}{Z_2} \right)$$

$$D = \frac{r}{Z_1} - \theta A_e \left(\frac{1}{Z_1} + \frac{1}{Z_2} \right) \quad (129)$$

Assume that r/Z_1 is the factor which is correct and that $\theta(A_e)(1/Z_1 + 1/Z_2)$ is causing D to deviate from zero. The following then can be said:

$$\theta = \frac{r}{Z_1} - \theta_0 A_e \left(\frac{1}{Z_1} + \frac{1}{Z_2} \right) \quad (130)$$

where θ_0 is the corrected value of θ . Subtracting Equation (130) from Equation (129) yields:

$$\theta_0 A_e \left(\frac{1}{Z_1} + \frac{1}{Z_2} \right) = D + \theta A_e \left(\frac{1}{Z_1} + \frac{1}{Z_2} \right)$$

or:

$$\theta_0 = \left\{ \frac{D}{A_e \left(\frac{1}{Z_1} + \frac{1}{Z_2} \right)} \right\} + \theta \quad (131)$$

From Equation (131) a new value of the tangents of α_1 and α_2 may be determined. If the difference of the tangents of these new angles differs from zero within the accuracy being carried out, then the author suggests the use of the following technique.

First, assume a straight line variation for the difference of the tangents of α_1 and α_2 at a proximity of the reflecting region of interest. Refer to Figure 4.

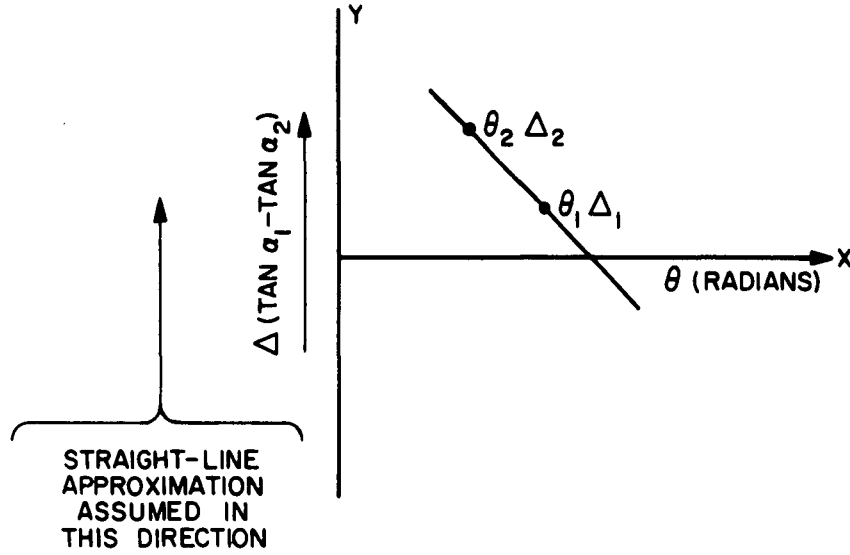


Figure 4. Straight Line Approximation of the Difference in the Tangents of α_1 and α_2 vs. the Angle θ .

In essence, the previous two equations have given the two noted points $\theta_2 \Delta_2$ and $\theta_1 \Delta_1$.

From theory stated previously, it is desired to have $\Delta = 0$ for absolute accuracy.

Hence, where M is the slope of the line and B the 0 intercept:

$$0 = M\theta + B \quad (132)$$

where

$$M = \frac{(\Delta_1 - \Delta_2)}{(\theta_1 - \theta_2)}$$

$$B = \Delta_2 - M\theta_2$$

Solving for the value of $\theta = \theta_0$ which will make $\alpha_1 - \alpha_2 = 0$ yields:

$$\theta_0 = -\frac{\Delta_2(\theta_2 - \theta_1)}{\Delta_2 - \Delta_1} + \theta_2 \quad (133)$$

where in general:

$$\Delta_2 = (\tan \alpha_1 - \tan \alpha_2) \text{ for the flat earth approximation.}$$

$\Delta_1 = (\tan \alpha_1 - \tan \alpha_2)$ for the incremented flat earth approximation or from Equation (131).

θ_1 = the value of θ obtained from the flat earth approximation.

θ_2 = the value of θ obtained from the incremented flat earth approximation or from Equation (131).

The above procedure may be repeated until the desired degree of accuracy is obtained by making the appropriate substitutions for Δ_1 , Δ_2 , θ_1 and θ_2 for each pass through the approximations. Theoretically, any desired degree of accuracy may be obtained. In practice, the author has found the above procedure to converge very rapidly. In general, within a range of three hundred meters and with altitudes above a few meters, the values of the two tangents can be brought to ten to twelve decimal places of accuracy by no more than six repeated approximations of Equation (133). For values of Z_1 and Z_2 above two thousand feet the convergence appears much more rapidly.

The preceding equations, therefore, provide a method within the accuracy of the geometrical framework, by which ΔR may be determined to within any theoretical desired accuracy ranging from the flat earth assumption to the exact equations. Admittedly, the labor involved increases with the desired accuracy. But, seldom are the flat earth approximations within the realm of reality for a physical situation, especially when the transmitted energy in the interference region is of a wavelength in the vicinity of ten to one hundred centimeters. This is especially true for small values of Z_2 . Also obtained has been an increase in the accuracy of ΔR for a specified number of significant digits carried in the calculation. For all calculations carried out in meters, the analysis of ΔR by the method outlined under the heading of the cubic in r_1 will require twenty decimal digits of accuracy to maintain ΔR accurate to the nearest one-hundredth of a meter within the interference region previously covered. The author's approach requires approximately twelve decimal digits of accuracy to maintain the same information. The author's approach does not make any assumptions such as are made in the evaluation of the cubic in r_1 , hence it is inherently more accurate.

CHAPTER IV

PROPAGATION FACTOR COMPONENTS OTHER THAN ΔR

Having spent so much effort in obtaining ΔR , which was previously shown to be a component of F , one might reasonably wonder if it might not have been better to find out what other components made up the factor F . If they proved to demand a multiplicity of the work previously described, then the analysis of ΔR might have been over-refined. The overall validity of the factor F can be no better than its weakest constituent. Luckily, however, the determining of ΔR is the major stumbling block in finding the component parts of F .

A. The Complex Coefficient of Reflection

Equation (41) indicates that a second factor which must be determined is $\bar{\Gamma}$. This factor takes into account the reflections of waves from the earth's surface. The reflection factor was previously defined as:

$$\bar{\Gamma} = \rho \epsilon^{-i\phi}$$

If the electromagnetic waves incident to the reflector are horizontally polarized, then $\bar{\Gamma}_h$ is defined as the ratio of reflected to incident electric field or the ratio of the vertical components of the magnetic field. The ratio of reflected to incident horizontal components of the magnetic field is $-\bar{\Gamma}_h$. For vertically polarized waves $\bar{\Gamma}_v$ is defined as the ratio of reflected to incident magnetic field. The ratio of the vertical components of the electric field or minus the ratio of the horizontal components is likewise equal to $\bar{\Gamma}_v$.

From these definitions, Fresnel's equation for the smooth plane earth can be derived:

$$\bar{\Gamma}_v = \rho_v \epsilon^{-i\phi_v} = \frac{(K_1/K_0)^2 \sin \psi_2 - \sqrt{(K_1/K_0)^2 - \cos^2 \psi_2}}{(K_1/K_0)^2 \sin \psi_2 + \sqrt{(K_1/K_0)^2 - \cos^2 \psi_2}} \quad (134)$$

$$\bar{\Gamma}_h = \rho_h \epsilon^{-i\phi_h} = \frac{\sin \psi_2 - \sqrt{(K_1/K_0)^2 - \cos^2 \psi_2}}{\sin \psi_2 + \sqrt{(K_1/K_0)^2 - \cos^2 \psi_2}} \quad (135)$$

The author will not derive these equations here as they may be found in any standard work^[11]. A variable in these equations is \bar{K}_1 where \bar{K}_1^2 is defined as the propagation factor of the earth:

$$\bar{K}_1^2 = \omega^2 \mu \epsilon - j\omega \mu \sigma \quad (136)$$

where $j = \sqrt{-1}$, ω is the frequency of the propagating energy, μ the permeability of the earth expressed in henries per meter, σ the conductivity of the earth in ohms per meter and ϵ the complex dielectric constant of the earth in farads per meter. The properties of the earth are expressed as relative values.

The variable K_0 is the square root of the propagation factor for air which for the air at the surface of the earth will be assumed approximately equal to that of free space.

$$K_0^2 = \omega^2 \mu_0 \epsilon_0 \quad (137)$$

where μ_0 and ϵ_0 are the permeability and dielectric constants respectively for free space. The permeability of the earth can essentially be said to be equal to that of free space. Hence, in view of the above $(\bar{K}_1/K_0)^2$ can be written as follows:

$$(\bar{K}_1/K_0)^2 = \epsilon/\epsilon_0 - j\sigma/\omega\epsilon_0 = \epsilon_1 - j\epsilon_2 \equiv \bar{\epsilon}_c \quad (138)$$

The quantity ϵ/ϵ_0 is the usual dielectric quantity commonly listed in tables for dielectric materials and $\bar{\epsilon}_c$ is called the complex dielectric constant. In terms of the MKS system of units:

$$\epsilon_2 = \sigma/\omega\epsilon_0 = 60\lambda\sigma \quad (139)$$

[11] J. A. Stratton, Electromagnetic Theory, McGraw-Hill, New York, 1941, Secs. 9.4 and 9.9.

where λ is the wavelength of propagation expressed in meters. Hence, $(\bar{K}_1/K_0)^2$ can be simplified to:

$$\bar{\epsilon}_c = \epsilon_1 - j60\lambda\sigma \quad (140)$$

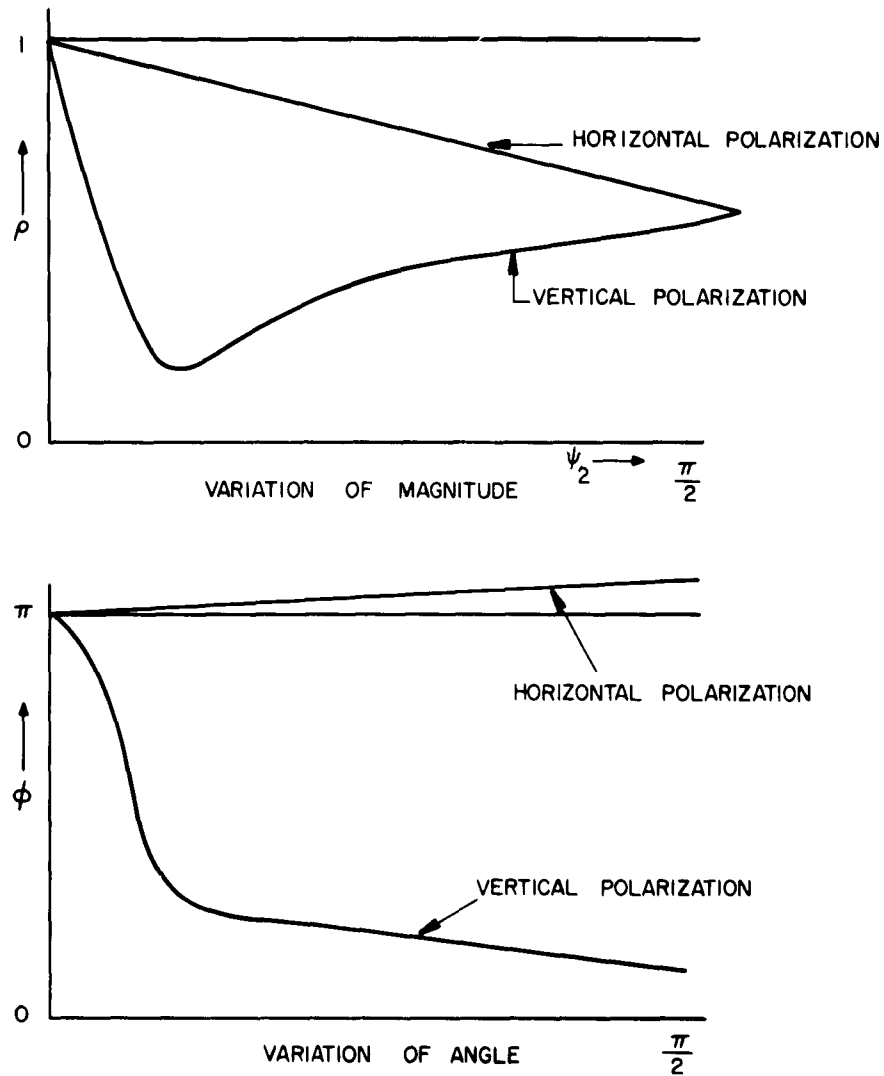


Figure 5. Variations of Complex Coefficient of Reflection With Incident Angle ψ_2 .

In general, the expected behavior of Equations (134) and (135) can best be described with few words and the use of Figure 5.

B. The Divergence Factor

A third factor which must be determined is D as seen from Equation (41). This factor takes into account that the electromagnetic energy is weakened upon reflection from a spherical earth by a divergence of the concentration of the energy. The

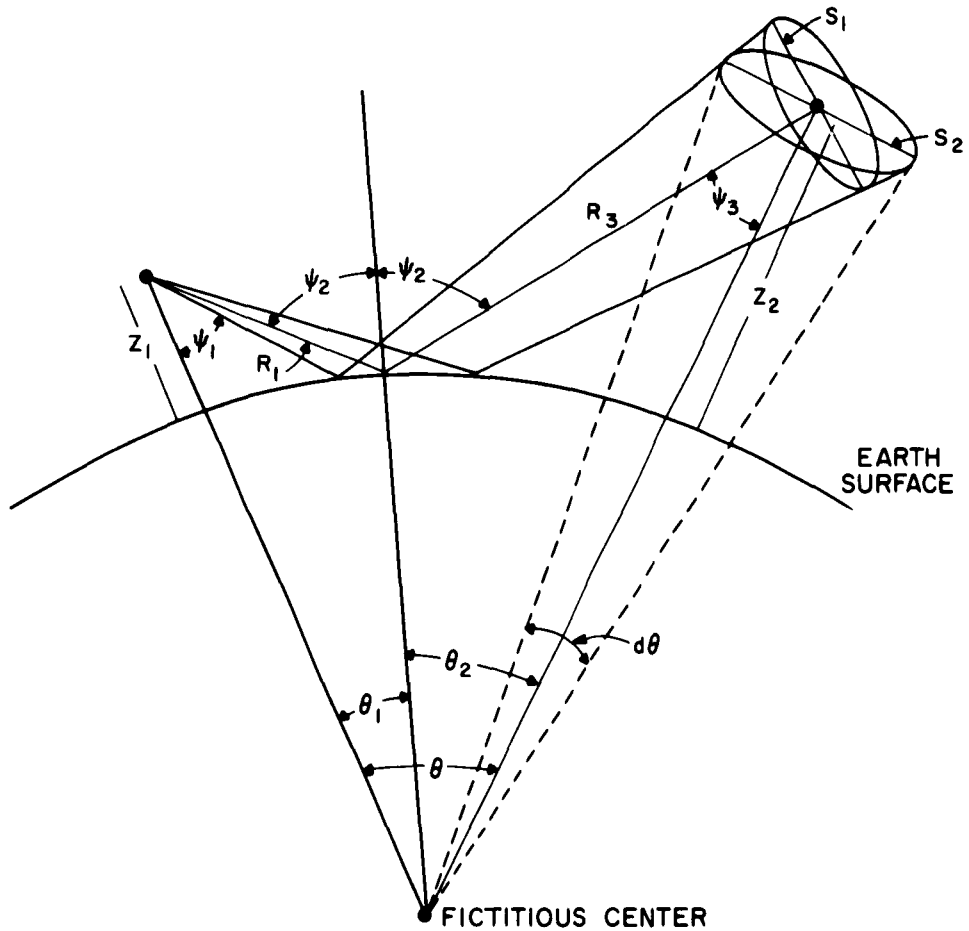


Figure 6. Curved Earth Geometry for Computation of the Divergence Factor

following analysis will help to clarify the general concept involved. Reference should be made to Figure 6. This analysis was first carried out in detail by van der Pol and Bremmer^[12] in a detailed analysis of defraction of waves by a spherical conductor.

The concept is based upon the principle of comparing the density of the reflected rays from the earth or sphere with those that would have been present had reflection occurred from a plane reflector. The field strength difference is proportional to the square root of the ray intensity difference. The analysis is carried out by comparing the cross section of a cone obtained from a spherical reflection with that obtained from reflection from a flat conductor.

The rays leaving the transmitting antenna have an effective cross-sectional area of:

$$R_s^2 \sin \psi_1 d\psi_1 d\alpha \quad (141)$$

where α is measured in a plane perpendicular to the plane of paper and R_s is the straight line distance from the antenna at Z_1 to the antenna at Z_2 . Reflection from a plane earth would result in an equivalent cross-sectional area of rays equal to:

$$(R_1 + R_3)^2 \sin \psi_1 d\psi_1 d\alpha \quad (142)$$

This is because the reflective path length appears from an image directly below the transmitting antenna at a point $-Z_1$. The cross-sectional area of the rays in the spherical case is s_1 or $s_2 \cos \psi_3$. From the geometry, s_2 may be written as $(A_e + Z_2) \sin \theta d\theta d\alpha$. Hence, the ratio of the cross-sectional areas may be expressed as:

$$\gamma = \frac{(R_1 + R_3)^2 \sin \psi_1 d\psi_1 d\alpha}{(A_e + Z_2)^2 \sin \theta \cos \psi_3 d\theta d\alpha} \quad (143)$$

[12] B. van der Pol and H. Bremmer, "Further Note on the Propagation of Radio Wave Over a Finitely Conducting Spherical Earth," Phil. Mag., No. 182, March 1939.

Or, the divergence factor is equal to:

$$D = \sqrt{\gamma} = \frac{R_1 + R_3}{A_e + Z_2} \sqrt{\frac{\sin \psi_1 d\psi_1}{\sin \theta \cos \psi_3 d\theta}} \quad (144)$$

Expressing $d\theta$ in terms of ψ_1 and θ permits D to be expressed in the following convenient manner. Reference may be made to the original works for the details involved.

$$D = \frac{A_e(R_1 + R_2) \sqrt{\sin \psi_2 \cos \psi_2}}{\sqrt{\left\{ (A_e + Z_2)R_1 \cos \psi_3 + (A_e + Z_1)R_3 \cos \psi_1 \right\} (A_e + Z_1)(A_e + Z_2) \sin \theta}} \quad (145)$$

Equation (145) is a function of the variables previously expressed as desirable. While it may be slightly unwieldy, the author has chosen to leave it in its present form so as to be of the same order of exactness as the previously derived expressions for ΔR . In general, D is seen to have its smallest value when the reflected energy approaches a grazing angle of incidence. As ψ_2 decreases, D will approach unity.

C. The Implementing Expressions

It now remains for the functions which implement the previous expression to be derived. Referring again to Figure 1, the tangent line designated as L is needed to determine when the functions are valid. By this it is meant that when Z_2 is located within the interference region or above line-of-sight of the transmitter, then the previous equations are valid. If the value of Z_2 as combined with a value of R_2 places the receiver or target out of sight of the transmitter, then the equations are not valid. Note that a line-of-sight correspondence between the two is also a degenerate situation. For a given value of transmitter height, the distance to the horizon may be found in the following manner. From Figure 1, it will be seen that a radius vector drawn from the center of the spherical earth intersects the tangent ray from a height Z_1 such that the angle between these two is equal to ninety degrees. Hence, the vector whose magnitude equals the radius of the earth, the tangent ray, and the vector from the center of the

earth to the transmitter form a right triangle. Therefore, the distance from the transmitter to the horizon can be expressed in the following manner:

$$d_h = \sqrt{(Z_1 + A_e)^2 - A_e^2} \quad (146)$$

If the value of R_2 , an independent variable, is greater than d_h , it is required to know at what height Z_2 , the receiver or target, must be in order that it falls into the interference region or where the equations are valid. Consider the limiting case where R_2 is at some point tangent to the surface of the earth. Call the distance from the point of tangency to the projected vector from the center of the earth to the receiver or target, d_1 :

$$d_1 = R_2 - d_h \quad (147)$$

A radius vector from the earth's center to the tangent point is again at right angles to d_h or $(R_3 - d_h)$. Hence, the intersection of the vector from the earth's center to the receiver by the vector d_1 occurs at a magnitude of the target-directed vector equal to d_2 :

$$d_2 = A_e^2 + d_1^2$$

The minimum height Z_2 in the interference region is, therefore:

$$Z_{2\min} \cong -A_e + \sqrt{A_e^2 + d_1^2} \quad (148)$$

In order to specify $f(\phi_1)$ and $f(\phi_2)$ as a function of R_2 , Z_1 , and Z_2 , the following analysis may be carried out in reference to Figure 7 which is taken directly from Figure 1. The analysis involves first the derivation of δ . From an oblique triangle consideration define a variable T and U as follows:

$$T = \frac{A_e + Z_2 + A_e + R_3}{2} = \frac{2A_e + R_3 + Z_2}{2} \quad (149)$$

$$U = \sqrt{\frac{(T - R_3)(T - A_e)(T - A_e - Z_2)}{T}} \quad (150)$$

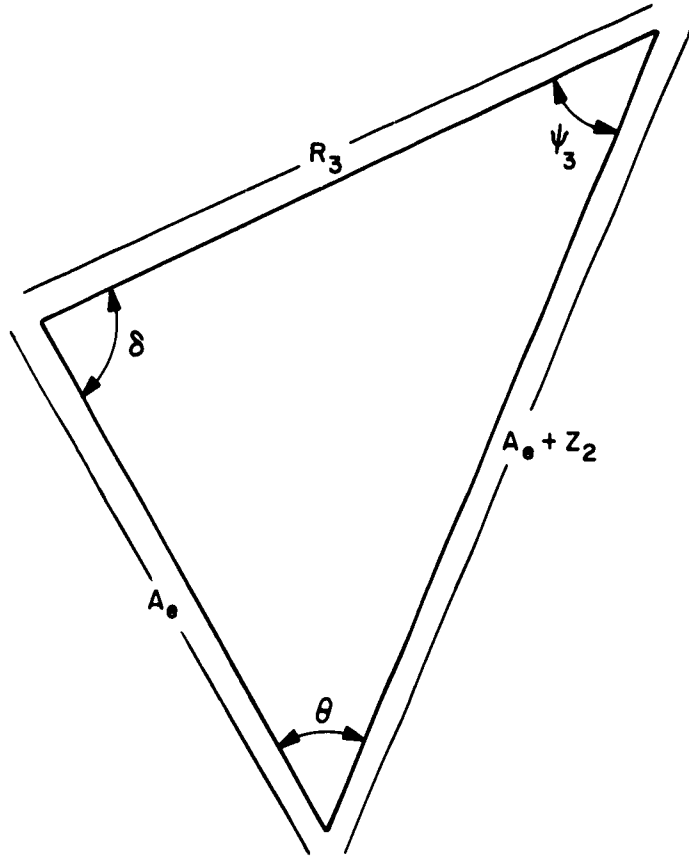


Figure 7. Excerpt from Figure 1 Used in the Derivation of δ .

Hence from the preceding:

$$\begin{aligned}
 \delta &= 2 \tan^{-1} \frac{U}{T - A_e - Z_2} \\
 &= 2 \tan^{-1} \sqrt{\frac{(T - R_3)(T - A_e)(T - A_e - Z_2)}{T(T - A_e - Z_2)^2}} \\
 \delta &= 2 \tan^{-1} \sqrt{\frac{(T - R_3)(T - A_e)}{T(T - A_e - Z_2)}}
 \end{aligned} \tag{151}$$

Hence, Equations (149) and (151) serve to solve for δ , having previously determined R_3 . The following relationships follow directly from Figure 1:

$$\psi_3 = \pi - \delta - \theta \quad (152)$$

$$\psi_4 = \pi - \delta - \theta_1 \quad (153)$$

$$\psi_2 = \frac{\pi}{2} - \alpha_1 = \frac{\pi}{2} - \alpha_2 \quad (154)$$

$$\phi_2 = -\left(\frac{\pi}{2} - \psi_4\right) \quad (155)$$

The value of γ in Figure 1 is determined in a manner similar to Equation (151):

$$\gamma = 2 \tan^{-1} \sqrt{\frac{(S - R_2)(S - R_1)}{S(S - R_3)}} \quad (156)$$

where

$$S = \frac{R_1 + R_2 + R_3}{2}$$

Hence $\phi_1 = \gamma - |\phi_2|$

$$\phi_1 = 2 \tan^{-1} \sqrt{\frac{(S - R_2)(S - R_1)}{S(S - R_3)}} - |\phi_2| \quad (157)$$

Within the preceding framework are the capabilities of analyzing an airborne-type transmitter with respect to its pattern of coverage. The analysis has been carried out by the method of ray analysis of the electromagnetic energy leaving the transmitting antenna. The equations are, of course, subject to their noted limitations.

CHAPTER V

LIMITATIONS OF THE ANALYSIS

At the start of this paper, the author mentioned that all equations were based upon the ability to represent electromagnetic waves by straight lines or rays. The author also indicated that this assumption may not always be exact or true. In the previous analysis, it was assumed that at all times the rays appear as normals to the surfaces of constant phase of the electromagnetic wavefronts. However, in the vicinity of sharp corners and in regions where the rays exhibit odd properties because of refraction, such as at a cusp, then the energy can no longer be said to follow the rays and geometrical optics fail to give results which are truly meaningful. At this point, it is required to analyze the energy by introducing the field of physical optics. Physical optics is not the theme of this paper but the following analysis should indicate to the reader where the previous analysis does break down and if it is important in the general area of pattern coverage. Also covered in this section will be surface roughness and the effect of the index of refraction of the atmosphere.

A. The Equation of the Ray

Geometrical optics may be invoked to analyze the situation where the atmosphere is non-homogeneous. In a homogeneous atmosphere, as previously assumed, the rays indeed represent the wavefronts and their direction of travel. In any atmosphere, Maxwell's equations apply, and hence they will be stated for a desired end result:

$$\nabla \cdot \bar{D} = \rho \quad (158)$$

$$\nabla \cdot \bar{B} = 0 \quad (159)$$

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \quad (160)$$

$$\nabla \times \bar{H} = \sigma \bar{E} + \frac{\partial \bar{D}}{\partial t} \quad (161)$$

$$\bar{B} = \mu \bar{H} \quad (162)$$

$$\bar{D} = \epsilon \bar{E} \quad (163)$$

The parameters are defined as follows: D is the electric flux density, ρ is the charge density at the point in question, B is the magnetic flux density, E is the electric field intensity, H is the magnetic field intensity, μ is the permeability, and ϵ the dielectric constant. In free space σ equals zero and μ and ϵ are not a function of time, and hence:

$$\nabla \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t} \quad (164)$$

$$\nabla \times \bar{H} = \epsilon \frac{\partial \bar{E}}{\partial t} \quad (165)$$

Then, the wave equations of free space may be determined:

$$\nabla \times (\nabla \times \bar{E}) = -\mu \nabla \times \frac{\partial \bar{H}}{\partial t}$$

$$\nabla(\nabla \cdot \bar{E}) - \nabla^2 \bar{E} = -\mu \frac{\partial}{\partial t} (\nabla \times \bar{H})$$

But:

$$\nabla(\nabla \cdot \bar{E}) = 0$$

Therefore:

$$\nabla^2 \bar{E} = \mu \epsilon \frac{\partial^2 \bar{E}}{\partial t^2} \quad (166)$$

In a similar manner:

$$\nabla^2 \bar{H} = \mu \epsilon \frac{\partial^2 \bar{H}}{\partial t^2} \quad (167)$$

Equations (166) and (167) are the wave equations in free space. If we assume that all of the propagated energy can be represented in a sinusoidal form, then $e^{j\omega t}$ can be used to represent the time variations of all fields and currents involved. The second time derivative can hence be represented by $-\omega^2$ where ω is the radian frequency of the propagated wave:

$$\nabla^2 \bar{E} = \mu \epsilon \omega^2 \bar{E} \quad (168)$$

or:

$$\nabla^2 \bar{H} = \mu \epsilon \omega^2 \bar{H} \quad (169)$$

It now behooves us to find a solution to these equations. In general, the solution may be sought in the following form. Either Equation (168) or (169) may be considered. Considering Equation (169), the solution is:

$$\bar{H} = A e^{j\omega S} \quad (170)$$

A and S are real functions of position. If Equation (170) is substituted into Equation (169) and the real and imaginary parts are equated to zero, the following results:

$$(\nabla S)^2 - \frac{\nabla^2 A}{A \omega^2} - \eta^2 = 0 \quad (171)$$

$$\nabla^2 S + \frac{2(\nabla S \cdot \nabla A)}{A} = 0 \quad (172)$$

where:

$$\eta^2 = \mu \epsilon$$

If ω^2 is quite large, then Equation (171) may be written as:

$$(\nabla S)^2 = \eta^2 \quad (173)$$

The solution to Equation (173) is simply to find a function whose gradient of the magnitude, but not the direction, is given as η^2 . Hence, according to the type of wavefront to be represented, a surface may be chosen where at every point we assign a constant value of S, say S_0 . The gradient of S is perpendicular to S at each point in question and ∇S may be completely defined. To find another surface $S = S_0 + S$, perpendiculars to the defined surface may be constructed of lengths equal to $\nabla S / \eta$ where η is the value of the index of refraction at the point in question. The traces of the constructed perpendicular vectors then define the desired surface. Hence, this is the method of producing surfaces of constant S. As $\nabla S \rightarrow 0$ in the limit, Equation (173) is satisfied. The surfaces of constant S represent the electromagnetic wavefront. The perpendiculars represent the rays used in the previous derivations. The differential equation of the ray will now be determined.

$$\overrightarrow{T'T''} = \overrightarrow{RT''} - \overrightarrow{RT'} = d\mathbf{p} \left(\mathbf{u} - \frac{\nabla \eta \cdot \mathbf{u}}{\eta} \right) d\sigma \quad (175)$$

where:

$$d\sigma = \frac{dS}{\eta}$$

as $\mathbf{u} \times \mathbf{v} = \mathbf{t}$ and $\mathbf{t} \times \mathbf{v} = -\mathbf{u}$

$$t + dt = \frac{\overrightarrow{T'T''} \times \mathbf{v}}{|\overrightarrow{T'T''}|} = t + \frac{\nabla \eta \cdot \mathbf{u}}{\eta} d\sigma \quad (176)$$

hence:

$$\frac{dt}{d\sigma} = \frac{\nabla \eta \cdot \mathbf{u}}{\eta} (\mathbf{u}) \quad (177)$$

Equation (177) is the differential equation of the ray. While its derivation may leave something to be desired, the author refers the reader to Kerr^[13], which suggests the foregoing analysis. Equation (177) gives a method for obtaining a ray type analysis from a direct consideration of Maxwell's equations. Hence, knowing a point R on the energy wavefront, a point T' can be determined by choosing a value of $d\sigma$.

$$\overrightarrow{RT'} = t d\sigma \quad (178)$$

The direction of the ray at T' is equal to:

$$t(T') = t(R) + \frac{dt}{d\sigma} (R) d\sigma \quad (179)$$

As $d\sigma \rightarrow 0$, the segments so obtained approach the continuous ray.

B. The Effect of η

It is now interesting to look at the ray pattern for first a homogeneous free-space atmosphere and then the stratified atmosphere. For the homogeneous atmosphere, clearly $\nabla \eta$ is equal to zero. Hence from Equation (177) $dt/d\sigma$ is equal to zero. Hence from Equation (179) the rays then can be seen to be indeed straight lines.

[13] Kerr, op. cit., pp. 41-50.

However, for the time, assume that the physical earth conditions may be represented by an index of refraction which changes only in the height direction. If the earth and its atmosphere are represented by the conventional x, y, z coordinate system, then η is everywhere only in the z direction. It then follows from the ray equations that for this somewhat idealized case of a stratified atmosphere that the rays representing the correct energy wavefronts are curves lying in planes passing through the z axis. This can be better seen if the absolute value of Equation (177) is obtained:

$$\left| \frac{dt}{d\sigma} \right| = \frac{|\nabla \eta| \cdot u}{\eta} (u)$$

If $|\nabla \eta|$ is considered vertical, then $|\nabla \eta| \cdot u = |\nabla \eta|$. Also, if η is considered approximately unity then:

$$\frac{1}{R} = |\nabla \eta|$$

where R is the radius of curvature. Hence, for a constant gradient of η the rays are approximately arcs of circles.

C. The Modified Index and Effective A_e

Under these assumptions, the straight-line ray analysis is seen to fall apart. However, it is possible to introduce a new parameter called the modified index of refraction N. The author will mainly state the following results obtained from Kerr. A modified index of refraction is obtained by considering Snell's Law and the previously derived ray equations:

$$N \equiv \left(1 + \frac{Z}{A} \right) \eta \approx \eta + \frac{Z}{A} \quad (180)$$

as $\eta \approx 1$.

Where as η generally decreases with height Z, where A is the radius of the earth, the modified index N will be seen to increase with height. When η decreases linearly with height:

$$\eta = \eta_0 + \left(\frac{d\eta}{dZ} \right) Z \quad (181)$$

where the rays are represented by straight-line segments instead of segments of arcs. The previous is obtained by a consideration of Maxwell's equations, the ray tracing equations, and Snell's Law.

If $d\eta/dZ$ (Z) is small compared to η_0 , the value for free space, then:

$$N = \eta_0 \left(1 + \frac{Z}{A_e} \right) \quad (182)$$

where:

$$\frac{1}{A_e} = \frac{1}{\eta_0} \frac{d\eta}{dZ} + \frac{1}{A} \quad (183)$$

A represents an effective radius of the earth, and $\eta_0 \approx 1$. Measurements have shown that, with the exception of a few hundred feet close to the earth, the gradient of η is such that the effective radius of the earth is:

$$A_e = 4/3 \quad (\text{True Radius}) \quad (184)$$

Hence, through the utilization of an effective earth's radius, a linear profile for the index of refraction may be taken into account in the analysis of the interference region. For a profile other than that of the linear case, the preceding becomes a poor assumption. However, the conditions may be so varied from one physical situation to another that it is impractical to attempt to obtain any type of a general qualitative analysis. If the profile for the index of refraction cannot be assumed linear, then the analysis would fall under a special classification such as duct propagation.

D. Additional Limitations on a Ray Analysis

In the solution of the radar wave equation, the ray analysis was derived under the assumption that:

$$\frac{\nabla^2 A}{A\omega^2} \ll \eta^2 \quad (185)$$

Through the analysis of this inequality and Equation (142) it can be shown that Equation (185) implies the restriction that the index of refraction must not change appreciably in a distance equal to that of the wave length of propagation. This is the reason that as the frequency increases the assumption of Equation (185) becomes increasingly more accurate. For a ray analysis it has been found that the preceding is in general valid from physical measurements.

An analysis of Equation (142) also imposes the restriction on the ray analysis that the fractional change in the spacing, between the bundle of rays representing the traveling energy, must be small when compared to unity over a distance of one wave length. In essence it must be remembered that a single ray has no meaning, but rather the single ray is used to indicate the composite bundles of rays which in turn represent the energy wavefronts. Hence, in the analysis, this condition is violated whenever it assumes that a focus exists. In considering rays which are curved, this condition is also violated when the rays undergo a change in sign of their curvature. To analyze the conditions imposed by Equation (142) it becomes necessary to resort to physical rather than geometric optics.

The previous ray analysis was presented with the intent to stress that problem areas exist where a ray analysis in the interference region may lead to erroneous results and conclusions. The reader should hence keep these areas of validity, and the assumptions that are made in the analysis, in mind when using the theoretical geometrical equations derived for the interference region. The author did not analyze in detail these situations where the ray analysis leads one into a meaningless procedure. The many different situations which may arise should be given their own separate analysis utilizing physical optics or the wave equations and physical optics. For the remaining section of this paper, it will be assumed that the index of refraction changes linearly with height and that the wavefronts may be analyzed by the method of straight ray analysis, assuming the modified radius of the earth to take into account the physical curvature of the rays assuming a linear profile for η .

E. The Rough Reflector

At this point, it might be beneficial to analyze the effect of the reflective surface roughness. It has been previously assumed that the surface could be represented by an even or smooth boundary with no irregularities. Consider a roughness whose height is equal to h . Let ψ represent the angle of incidence. Reference may be made to Figure 9.

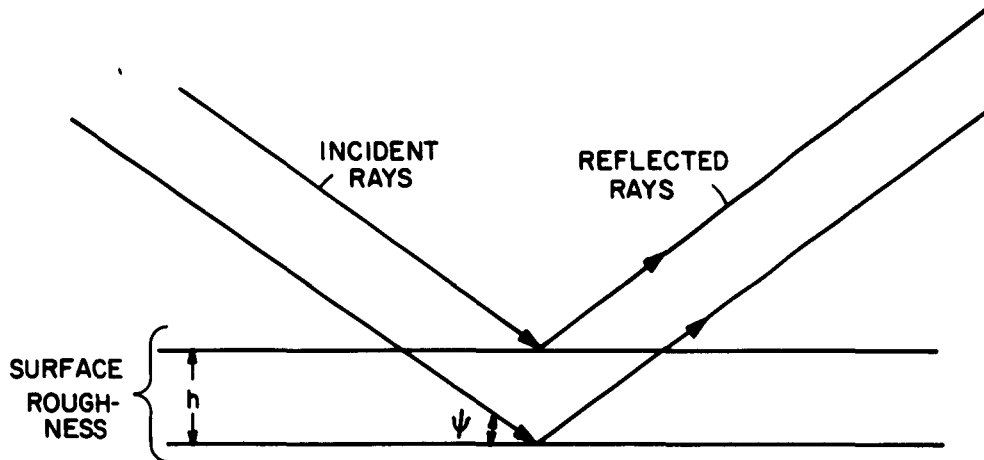


Figure 9. Phase Difference between Rays Reflected from Two Levels

The phase difference between the two rays is equal to:

$$\Delta R = 2 h \sin \psi \quad (186)$$

which corresponds to a phase difference of:

$$K \Delta R = \frac{4\pi h}{\lambda} \sin \psi \quad (187)$$

Hence, in terms of this phase difference, the surface roughness may be specified.

When the phase difference equals π , the surface is effectively its roughest. There is

no established criterion on just what constitutes a rough surface but the following may be chosen:

$$h \sin \psi < \frac{\lambda}{8} \quad (188)$$

If $h \sin \psi < \frac{\lambda}{8}$ this surface may be assumed smooth.

Needless to say, this is only a crude quantitative analysis of surface roughness. If the irregularity is of the order of a wave length, then the problem of refraction should be considered. Equation (188) does, however, emphasize that the reflected energy is less affected by a given height of roughness as ψ decreases or as λ increases. For a surface for which the roughness has to be represented as a statistical distribution of irregularities, the method of analysis is undeveloped at this time. Truly a ray analysis fails for this situation and at present electromagnetic wave theory has trouble handling the boundary conditions involved for a statistical distribution of surfaces. Hence, an analysis of this type at the present state-of-the-art is made by use of rough and simplified refraction theory or from direct physical observations and attempted correlation. However, very little may be said at the present time in terms of a definite correlation between roughness and reflection.

There has been some work done by assuming that the surface roughness can be represented by a series of ellipses within which a given phase retardation for reflection can be assumed. The basis of this assumption is that a single reflected ray is in essence representing radiation from the actually illuminated surface. In reality a large surface is illuminated by the incident energy. Currents at the surface are then induced which in turn cause radiation in all directions. At a specific point in space, the sum of the induced radiation components yields the reflected field. The area and dimension of each ellipse can be derived in terms of the wave length of radiation involved, such that the phase of the radiation from each ellipse is essentially constant. If the surface roughness is consequently larger than the calculated ellipse, then an analysis can be carried out by assuming essentially multiple reflection points in a ray analysis and making calculations based upon this assumption. The usefulness of this technique, in general, may be limited and hence the mathematical derivations have been excluded.

However, the reader should become aware of the surface roughness problem in a theoretical analysis. Even if the surface roughness over a given area of reflection in a physical situation could be specified, which is many times difficult, the analysis techniques are not well defined. A qualitative analysis for each physical situation involved may be made in respect to the preceding, but to attempt to obtain a quantitative type of analysis is almost hopeless in view of the parameters involved and the state-of-the-art. It appears that a theoretical analysis should be carried out by assuming a smooth earth and then modifying the results qualitatively should the roughness phenomenon appear to be prominent in the overall analysis.

F. Antenna Phase Pattern Effect

It might be interesting at this point to consider the situation where the energy radiated from the transmitting antenna is not of the same phase in the direction of ϕ_1 and ϕ_2 . From the previous analysis of F the following conclusion can be immediately drawn. If $f(\phi_1)$ or $f(\phi_2)$ are complex, then they can be represented as a complex number in the expression of F. Hence, the analysis as carried out in the derivations need to be only slightly modified. The effect of such circumstances would be difficult to analyze in general. However, one would expect that the shape of the lobes of coverage of the transmitting equipment would be changed. In other words, the area of maximum coverage could be governed in this fashion.

The feasibility of being able to specify a phase difference from the different parts or sections of the antenna is a different problem. So far as the author knows, no commercial producers of high-powered antennas specify a phase difference in radiation from various sectors of their antennas.

CHAPTER VI

A DIGITAL COMPUTER ANALYSIS

It is appropriate at this time to present a method which is applicable to finding the coverage diagram of a transmitter located over an assumed smooth surface and in an atmosphere which has a linear profile for the index of refraction. The type of transmission will be assumed as two way or the coverage diagrams obtained will be that for a radar. The method will involve the solution of the equations of Chapter VI (A) by a high-speed digital computer — specifically the IBM 704 or 709 or 7090. However, the resulting computer program should give considerable help to the reader in applying the techniques to any digital computer.

A. The Necessary Equations

The necessary equations to completely specify the problem and plot the coverage diagram are as follows. Reference should be made to Figure 1.

1. Equation (30) gives the free-space radar range equation:

$$R_0 = \sqrt[4]{\frac{W_t}{W_r} \frac{\sigma}{4\pi}} \sqrt{\frac{G\lambda}{4\pi}} \quad (30)$$

where G is the maximum gain of the transmitter, λ the wave length of the transmitted waves, W_t the peak power output of the transmitter in watts, W_r the minimum value of useful power to the receiver, and σ the target area in square meters.

2. The following equation is useful to give the maximum possible value of R_2 under ideal conditions:

$$R_{2\max} = 2R_0 \quad (189)$$

3. An empirical solution for the radius of the earth as obtained by Massachusetts Institute of Technology's Lincoln Laboratory will be used. This equation assumes an ellipsoidal earth:

$$A_e = 1852(3438.26815 + 577804 \cos(2L) + 0.01222 \cos(4L) + H_{rs}) \quad (190)$$

where L is the latitude in degrees of the area covered in reference to the geometric north pole, and H_{rs} is the height above sea level in meters of the area involved. A linear profile for the index of refraction will be assumed in all calculations, hence the effective earth radius will be $4/3$ the calculated value. The region of reflection will be assumed spherical after the above computation.

4. The two equations which indicate the boundary of the interference region will be employed. These equations previously appeared as Equations (146) and (148). The distance from the point Z_1 to the horizon is:

$$d_h = \sqrt{(Z_1 + A_e)^2 - A_e^2} \quad (146)$$

If R_2 is greater than d_h :

$$Z_{2\min} \geq -A_e + \sqrt{A_e^2 + d_1^2}$$

$$d_1 = R_2 - d_h$$

5. The value of the angle formed with the center of the earth as a function of Z_1 , Z_2 , and R_2 is a desired parameter. From Equation (125) the angle results:

$$\theta_1 = 2 \tan^{-1} \sqrt{\frac{(s - A_e - Z_1)(s - A_e - Z_2)}{s(s - R_2)}} \quad (125)$$

$$s = \frac{2A_e + R_2 + Z_1 + Z_2}{2}$$

6. The value of the tangent of α_1 and α_2 is needed to find the distances R_1 and R_3 . Equations (117) and (118) rewritten appear as:

$$\tan \alpha_1 = \frac{\sin(\theta_1 - \theta)}{\cos(\theta_1 - \theta) - R/(R + Z_1)} \quad (117)$$

$$\tan \alpha_2 = \frac{\sin \theta}{-\cos \theta + R/(R + Z_2)} \quad (118)$$

7. The values of R_1 and R_3 are hence given by Equations (121) and (122):

$$R_1 = \left\{ \frac{[(Z - 1) + (Z_1 \times Z)/A_e]}{\cos \alpha_1} \right\} A_e \quad (121)$$

where:

$$Z = \cos(\theta_1 - \theta)$$

$$R_3 = \left\{ \frac{[(\cos \theta - 1) + (Z_2 \cos \theta)/A_e]}{\cos \alpha_2} \right\} A_e \quad (122)$$

8. The value of δ is given by Equation (151) as:

$$\delta = 2 \tan^{-1} \sqrt{\frac{(T - R_3)(T - A_e)}{T(T - A_e - Z_2)}} \quad (151)$$

$$T = \frac{R_3 + 2A_e + Z_2}{2}$$

9. From Equation (151), the values of ψ_3 , ψ_4 , ψ_2 and ϕ_2 may be determined as:

$$\psi_3 = \pi - \delta - \theta \quad (152)$$

$$\psi_4 = \pi - \delta - \theta_1 \quad (153)$$

$$\psi_2 = \frac{\pi}{2} - \alpha_1 = \frac{\pi}{2} - \alpha_2 \quad (154)$$

$$\phi_2 = -\left(\frac{\pi}{2} - \psi_4\right) \quad (155)$$

10. The value of ϕ_1 is needed for completely specifying the transmitter functions and appears as Equation (157):

$$\phi_1 = 2 \tan^{-1} \sqrt{\frac{(S - R_2)(S - R_1)}{S(S - R_3)}} - |\phi_2| \quad (157)$$

$$S = \frac{R_1 + R_2 + R_3}{2}$$

11. The complex values of the coefficient of reflection are given by:

$$\bar{\Gamma}_v = \frac{\bar{\epsilon}_c \sin \psi_2 - \sqrt{\bar{\epsilon}_c - \cos^2 \psi_2}}{\bar{\epsilon}_c \sin \psi_2 + \sqrt{\bar{\epsilon}_c - \cos^2 \psi_2}} \quad (134)$$

$$\bar{\Gamma}_h = \frac{\sin \psi_2 - \sqrt{\bar{\epsilon}_c - \cos^2 \psi_2}}{\sin \psi_2 + \sqrt{\bar{\epsilon}_c - \cos^2 \psi_2}} \quad (135)$$

where (v) and (h) designate vertical and horizontal polarization respectively and:

$\bar{\epsilon}_c$ equals $\epsilon_1 - j60\lambda\sigma$, with units as previously defined.

12. The divergence of energy from a curved surface is taken into account by D:

$$D = \frac{A_e(R_1 + R_3) \sqrt{\sin \alpha_1 \cos \alpha_1}}{\sqrt{\{(A_e + Z_2)R_1 \cos \psi_3 + (A_e + Z_1)R_3 \cos \psi_4\} (A_e + Z_1)(A_e + Z_2) \sin \theta_1}} \quad (145)$$

13. From Equations (32) and (41), the propagation factor F is given by:

$$F = \left| \left[f(\phi_1) + f(\phi_2) \Gamma D \epsilon^{-j\gamma} \right] \right| \quad (191)$$

where:

$$\gamma = (\Delta R) \frac{2\pi}{\lambda} \quad (192)$$

$$\Delta R = R_1 + R_3 - R_2 \cong 0 \quad (193)$$

$$\epsilon^{-j\gamma} = \cos \gamma - j \sin \gamma \quad (194)$$

14. It will be found convenient to specify a quantity K_2 as:

$$K_2 = \frac{\frac{.5}{R_2}}{\frac{2R_0}{R_2}} = \frac{R_0}{R_2} \quad (195)$$

From Equation (28), the power received by the radar receiver is equal to:

$$W_{r1} = \frac{G^2 \lambda^2 \sigma F^4}{(4\pi)^3 R_2^4} W_t \quad (196)$$

with units as previously defined. Utilizing Equation (30):

$$K_2 = \frac{R_0}{R_2} = \frac{\sqrt[4]{\frac{W_t \sigma}{W_r 4\pi}} \sqrt{\frac{G\lambda}{4\pi}}}{R_2} \quad (197)$$

Therefore:

$$\begin{aligned}
 |K_2 F|^4 &= \frac{W_t}{W_r} \frac{\sigma}{4\pi} \frac{G^2 \lambda^2}{(4\pi)^2} \frac{F^4}{R_2^4} \\
 &= \frac{W_t}{W_r} \frac{F^4 G^2 \lambda^2 \sigma}{(4\pi)^3 R_2^4} \\
 &= \frac{W_{r1}}{W_r}
 \end{aligned} \tag{198}$$

Hence, from Equation (198), the target as a function of Z_1 , Z_2 , and R_2 is within the pattern of coverage of the transmitter if:

$$|K_2 F|^4 \geq 1.0 \tag{199}$$

The target is outside the area of coverage of the receiver if:

$$|K_2 F|^4 < 1.0 \tag{200}$$

Hence, the pattern of coverage is determined by assuming a fictitious target over a suitable grid representing the interference region and then determining at each point if Equation (199) or (200) prevails.

The preceding equations have been chosen because they are advantageous for the purpose of plotting a coverage diagram. First, any desired degree of accuracy may be maintained. Secondly, they are functions of the parameters used in a coverage analysis. Thirdly, they represent, in as far as is feasible, the actual situation involved. True, some situations have been idealized. However, the composite relations are worthy of a detailed analysis because they give an insight into the various relations involved. In many cases, they approach reasonably well the practical situation. This is especially true for an area over the ocean. This is because the surface of the sea does not differ from the theoretical as greatly as a terrain. Certainly, they provide at their worst,

basic building blocks upon which the coverage analysis may rest. This is becoming increasingly important in establishing high frequency criteria for design.

B. General Computation Logic

The general logic used to solve the previous equations is to digitally trace the pattern of coverage as formed in space. The logic first assumes some starting value of range R_2 and target height Z_2 . At this set value of R_2 , the value of Z_2 is reduced until the boundary condition between Equations (199) and (200) is satisfied. The programmer must indicate whether this first point is on the top or bottom of an assumed lobe-type coverage pattern. The stated necessary equations must be solved to test for each point in space considered. When for a set value of R_2 the correct value of Z_2 is determined, then a tabulation of the variables is made and R_2 is increased or decreased according to the point being on the bottom or the top of a lobe. The maximums and minimums of the coverage pattern are tested by ΔR changes and by changes in the sign of the difference between two successive power calculations by Equation (198). Practically, the logic is as shown in Figure 10.

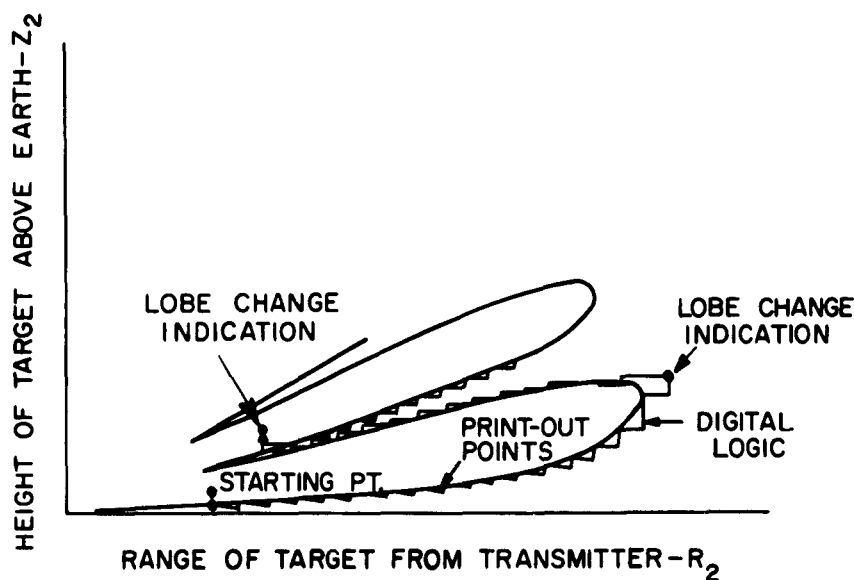


Figure 10. Pictorial Logic of Pattern Tracing

Much time has been devoted to applying and perfecting the rough outline of the logic previously presented. Its many obscure points require a thesis for elaboration. A rough outline of the program written by the author will be presented. Reference to the particular sections of the Fortran listing should be made to pages 73 through 87. The Fortran logic is a working logic and has been used on an IBM 7090. Some of the results may be found in the appendices. The following is a list of the major program symbols in the terms of symbols used in this paper:

AE	=	radius of earth
ARTHO	=	horizon range - L or d_h
Z2US	=	Z_2
RAN	=	R_2
ANG(2)	=	θ
ANG	=	θ_1
TAG1	=	$\text{TAN } \alpha_1$
TAG2	=	$\text{TAN } \alpha_2$
RANG1	=	R_1
RANG3	=	R_3
RANG13	=	ΔR
APHO2	=	α_2
APHO1	=	α_1
GMR	=	$\text{Re } (\Gamma)$
GMI	=	$\text{Im } (\Gamma)$
DE	=	D
SI2	=	ψ_2
SI3	=	ψ_3
SI4	=	ψ_4
TATA1	=	ϕ_1
TATA2	=	ϕ_2
ATA	=	δ
FELD1	=	F

CALL () indicates a double precision routine

DPREAD	-				Double-precision read of Data Cards
DPADD	-	"	"	+	
DPMPY	-	"	"	x	
DPROOT	-	"	"	$\sqrt{\quad}$	
DPSUB	-	"	"	-	
DPDIV	-	"	"	\div	
ASDP	-	"	"	SIN^{-1}	
DPSIN	-	"	"	SIN	
DPCOS	-	"	"	COS	

Referring to page 73, Section A of the Fortran listing is a prologue to the main program. It assigns storage space for double-precision quantities which are used in the program. Double-precision operations maintain at least fourteen significant decimal digits. These are required at points previously noted. The general accuracy of a digital computer the size of the IBM 7090 is seven significant decimal digits. Note that all sections of the Fortran listing precede the symbol.

Section B of the program reads in data stored on punched cards. Data Card 1 must contain in order of their appearance in fields:

- H - the maximum gain of the transmitting antenna.
- B - the wave length of propagated wave in meters.
- D - the peak output power of the transmitter in watts.
- E - the minimum useful signal to the receiver in watts.
- F - the effective target area in MKS units.

This first card must consist of five data fields; each field is ten columns wide. The data are entered in the floating point mode. In each field, the following must hold:

- Column 1 - sign of fraction.
- Column 2 - optional zero punch.
- Column 3 - the decimal point.
- Columns 4-9 - the quantity expressed as a decimal.
- Column 10 - the alpha-symbol E.

Column 11 - the sign of the decimal exponent.

Columns 12-13 - the power-of-ten decimal multiplier.

Data Cards 2 through 27 must contain the value of the antenna pattern stored at intervals of one degree in elevation from +90 degrees to -90 degrees. Starting with the first card, seven values of the antenna pattern function are stored at one degree increments starting at plus ninety degrees. The field width of each value of $f(\phi)$ is ten columns. The magnitude of $f(\phi)$ must be stored as a three digit decimal which must occupy columns four through six of each field. The remaining columns of each field are utilized as explained before. Data Card 28 must contain in order of the fields:

- Q - the relative dielectric constant of the reflecting surface.
- R - the conductivity of the reflecting surface.
- GAM - the value 1.0 if horizontal polarization is being considered and the value 0.0 if vertical polarization is being considered.
- AZ - the azimuth heading for which the 180 degrees values of $f(\phi)$ are stored.
- TA - the value +1.0 to test lobe logic at start of run or the value -1.0 if the test is not desired.
- ERD - the value of production block printouts desired per page.
- SLZ2US - the slope of the assumed starting lobe as referenced to rectangular coordinates.

The format of the data is again as a decimal to the desired power of ten in fields ten columns wide. Data Card 29 has the following information:

- GMDO - has the value of -1.0 if the principal root is to be used in determining Γ . The value +1.0 utilizes the program as written.
- STRAN - the minimum value of R_2 allowed in program.
- RBCC - a value of - .5 allows for the grid to hunt over an area in terms of the propagated wave length equal to one half of a revolution before indicating the end of a lobe. A value greater than - .5 allows less area and a value less than - .5 allows for a greater area.
- PRV - allows for monitor printout to be printed with tabulated lobe data.

Data Card 30 consists of the following five quantities:

- G - latitude of reflection point from the north pole.
- ZAM - maximum value of Z_2 allowed in program.
- AIZ - the minimum value of Z_2 allowed in program.
- RWHY - the value of 2.0 starts the program over a lobe tracing logic assumed as the bottom of a lobe. The value of 0.0 assumes the starting point as the top of a lobe.
- EROE - the value of $4/3$ modifies A_e as assumed by a linear profile of η .

The next six data cards must contain double-precision quantities stored one per card.

They appear in the order of cards as:

- AMD - the maximum allowable difference in $\tan \alpha_1$ and $\tan \alpha_2$ relative to θ .
- P - the value of Z_1 .
- BET - ΔZ_2 .
- DEL - ΔR_2 .
- SRAN - starting $R_2 - \Delta R_2$.
- SZ2US - starting $Z_2 - 5 \times \Delta Z_2$.

The format of these cards is as follows:

- Column 1 - blank.
- Column 2 - optional zero punch.
- Column 3 - optional decimal point.
- Columns 4-20 - sixteen decimal digits.
- Column 21 - alpha-symbol E.
- Column 22 - the sign of the decimal exponent.
- Columns 23-24 - the decimal power of ten.

The statements immediately following statement 15 calculate R_0 , $2R_0$, and the radius of the earth by Equations (30), (129), and (190) respectively. The high order part of all double-precision data will also be seen to be set equal to zero. Statements 408 to the end of Section B give a prologue printout of the program parameters on tape and/or paper.

Section C consists of pre-setting the lobe-tracing logic.

Starting on page 75, Section D increments Z_2 and R_2 from their read-in starting values. R_2 is then compared to d_h or L as given in Equation (146). If R_2 is greater than d_h , Z_2 is then compared to $Z_{2\min}$ as calculated by Equation (148). After forcing Z_2 to satisfy this condition, program stops and error checks are tested. Optional printout routines will be noted. It should be noted that in Section D, statements 31 and 32 increment R_2 in the proper manner when this section is branched into from later sections of the program. Z_2 is likewise adjusted by statements 39 and 40.

A double-precision calculation of θ_1 is performed in Section E. Statements 478 minus 1 through statement 480 give an optional monitor of the critical parameters involved in this calculation.

Section F, covering some one hundred statements through statement 924, serves to find two values of θ which may then be used in the straight line approximation to make α_1 equal α_2 . Section F starts by finding a value of θ by the flat earth approximation. This appears as statements 830 through 831. It should be noted that throughout this and the previous section, the angles themselves have not been calculated. The sine and cosine of all angles are used directly. This procedure was used to increase speed while maintaining the desired accuracy throughout the program.

After having found a value of θ by the flat earth approximation, control is advanced some eighty statements to statement 130. Statements 130 through 145 calculate the values of the tangents of α_1 and α_2 knowing θ . Note that this is essentially a manipulation of Equations (117) and (118). Having made this calculation, control is transferred back to statement 832 and through to statement 901. Monitor action will be noted at this point. If the values of the two tangents are both positive, then a second value of θ is calculated by the incremented flat earth approximation or Equation (131) slightly modified. This may be seen in statements 833 to 833 plus 3 on page 78. The tangents of α_1 and α_2 are then calculated as before and monitor printout will be noted following statement 845.

At any time during the preceding two approximations should the angles become such that the tangents of α_1 or α_2 become negative, then the following logic is used. The section of the reflecting earth which is visible to both the transmitting platform and the assumed target is first determined. This section is then divided into four equal sections. Two values of θ are then derived which causes the reflection point to fall within two sectors which are not equal. Having two valid sets of data, the straight-line approximation is then used to find the true reflection point. If the previous method does not yield points which are valid, then the particular point in space under consideration is rejected and a point one increment in height is considered. This forcing procedure is contained in the statements on page 77.

Section G, statements 148 to 421, on pages 78 and 79 calculates the new value of θ by the straight-line approximation such that $\tan \alpha_1$ equals $\tan \alpha_2$. Optional monitoring printout will be noted as statements 481 through 484 plus 1. The validity of the calculations are again attested to by statements 495 and 936. Statement 937 determines whether the two tangents are equal to the precision desired. If they are not, the procedure is repeated starting with statement 148; if they are, the program continues. Statements 130 through 145 of Section F are used in the calculation of the tangents.

Section H on page 79 calculates α_1 , α_2 , ΔR , R_1 and R_3 by Equations (193), (121) and (122). Optional monitoring is again provided.

The parameters ψ_3 , ψ_4 , ψ_2 , θ_2 , α , Γ , and ϕ are calculated in Section I. The calculations are straightforward as appearing in this text. The polarization of Γ is chosen from a data card.

The divergence factor D is calculated by Equation (145) in Section J.

Section K on page 81 converts ΔR to its equivalent phase constant by Equation (192). The operations of Equation (194) are also performed.

Section L reads in from stored data the values of $f(\phi_1)$ and $f(\phi_2)$ and interpolates between the one degree stored increments.

Section M calculates F by Equation (191), K_2 by Equation (195) and $|K_2 F|$ by Equation (198).

Within Section N are the lobe-extremes detecting criteria. This section covers branching of the logic at lobe maximums and minimums.

Section O, starting on page 82 and ending on page 84, contains the lobe tracing and printout logic for points being considered as nearer the top of a lobe than to the bottom. This section decides whether the value of Z_2 and R_2 combined comprise a valid printout point and if not, to what part of the program control must be transferred. Z_2 and R_2 are increased or decreased according to the logical decision. This section makes use of Equations (199) and (200). Statements 171 to 611 on page 83 predict the new point on the lobe from previous information of valid printout points.

Section P on pages 84 to 86 is a duplicate of Section O for points on the bottom of a lobe.

The program error stop printouts and formats are contained in Section Q. The section also contains debugging sequences and the final program stop.

The fact that the Fortran listing is reduced in its print size does not indicate that it is to be taken for granted. Within its framework lies the answer to the many problem areas that arise in solving the equations necessary in plotting a theoretical coverage diagram. Due to a lack of space, the many fine points and problem areas have not been discussed but may be determined from the Fortran listing by the individual who is faced with a similar problem. It should be noted that a small digital computer would not contain the storage needed by the logic represented in the previous analysis. The program tested maintains an overall accuracy of at least three decimal digits when the wave length of the propagated waves is equal to or greater than one tenth of a meter.

C. Fortran Listing

```

DIMENSION AMD(2),P(2),BET(2),DEL(2),AE(2),ST1(2),SRAN(2)
DIMENSION ST2(2),ARTHO(2),ST3(2),ZZUS(2),RAN(2),FRED(2),C1(2)
DIMENSION ZZM(2),BB(2),C2(2),S(2),TM(2),TE(2),CD(2),C3(2),ANG(2)
DIMENSION SI1(2),SI(2),CO(2),Z(2),ZB(2),DA(2),TAG1(2)
DIMENSION DB(2),TAG2(2),FRY(2),RAY(2),AVA(2),TRY(2)
DIMENSION CELM2(2),DLIM2(2),APH01(2),SLOC(2)
DIMENSION RANG1(2),APH02(2),RANG3(2),RANG13(2),A(180),SZZUS(2)
DIMENSION VSANG(2),COANG(2),AEP(2),AEZ(2),SIANG(2)
COMMON SLOC
SENSE LIGHT 0
PRINT 98
98 FORMAT(1H1/61H SENSE SWITCH 0,1,0,1,0,1***PRESS START AND CONTINU
  XE PROGRAM)
  PAUSE
  IF ACCUMULATOR OVERFLOW 7,7
7 IF QUOTIENT OVERFLOW 6,6
6 IF DIVIDE CHECK 1,1
1 READ INPUT TAPE 2,2,H,B,D,E,F
  READ INPUT TAPE 2,361,(A(J),J=1,180)
  READ INPUT TAPE 2,250,Q,R,GAM,AZ,TA,ERD,SLZ2US
  READ INPUT TAPE 2,628,GMD0,STRAN,RBCC,PRV
  READ INPUT TAPE 2,4,G,ZAM,AIZ,RWHY,EROE
  READ INPUT TAPE 2,3,AMD(1),P(1),BET(1),DEL(1),SRAN(1),SZZUS(1)
  IF(ZAM)9,9,15
9 READ 2,H,B,D,E,F
2 FORMAT(5E13.6)
  READ 361,(A(J),J=1,180)
361 FORMAT(7E10.3)
  READ 250,Q,R,GAM,AZ,TA,ERD,SLZ2US
250 FORMAT(7E10.3)
  READ 628,GMD0,STRAN,RBCC,PRV
628 FORMAT(4E17.9)
  READ 4,G,ZAM,AIZ,RWHY,EROE
4 FORMAT(E12.5,2E17.9,2E1.5)
  READ 3,AMD(1),P(1),BET(1),DEL(1),SRAN(1),SZZUS(1)
3 FORMAT(E24.9)
15 C=4.0*3.14159265
  AB=(H*B)/C
  AC=(D/E)*(F/C)
  AD=SQRTF(AC)
  RANGE=(SQRTF(AB))*(SQRTF(AD))
  RANGM=2.0*RANGE
  AE(1)=1852.0*(3438.2682-5.77804*(COSF(2.*G))-0.01222*(COSF(4.*G)))
  AE(2)=0.0
  CALL DPADD(AE,AE,ST1)
  CALL DPADD(ST1,P,ST2)
  CALL DPMFY(ST2,P,ST1)
  CALL DPR00T(ST1,ARTHO)
  SAE=AE(1)
  AE(1)=AE(1)*EROE
  P(2)=0.0
  AMD(2)=0.0
  BET(2)=0.0

```

```

      DEL (2)=0.0
      NO=0
      SRAN (2)=0.0
      SZZUS(2)=0.0
      IF(SENSE SWITCH 1)408,409
408 PRINT 401
401 FORMAT(55H          LOBE PATTERN PROPAGATION TRACE)
      PRINT 402,AZ,ARTHO(1)
402 FORMAT(17HOAZIMUTH HEADING=E10.3,30H LINE OF SIGHT OF TRANSMITTER=
      XE17.9)
      PRINT 407,AE(1),RANGE,RANGM,SZZUS(1)
407 FORMAT(4HOAE=E17.9,7H RANGE=E17.9,7H RANGM=E17.9,17H STARTING HEIG
      XHT=E17.9)
      PRINT 404,H,B,D,E,F
404 FORMAT(10HOMAX GAIN=E13.6,13H WAVE LENGTH=E13.6,10H PEAK PWR=E13.6
      X,9H MIN SIG=E13.6,13H TARGET AREA=E13.6)
      PRINT 405,G,ZAM,Q,R
405 FORMAT(24HOLATITUDE FROM NTH POLE=E12.5,12H MAX TGT HT=E17.9,19H R
      XEL DIEL CONSTANT=E10.3,14H CONDUCTIVITY=E10.3)
      PRINT 406,P(1),BET(1),DEL(1),SRAN(1),AMD(1)
406 FORMAT(12HOPLTFORM HT=E17.9,10H DELT ALT=E10.3,10H DELT RAN=E12.5,
      X14H STARTING RAN=E17.9,5H AMD=E10.3/1H1)
409 WRITE OUTPUT TAPE 7,401
      WRITE OUTPUT TAPE 7,402,AZ,ARTHO(1)
      WRITE OUTPUT TAPE 7,407,AE(1),RANGE,RANGM,SZZUS(1)
      WRITE OUTPUT TAPE 7,404,H,B,D,E,F
      WRITE OUTPUT TAPE 7,405,G,ZAM,Q,R
      WRITE OUTPUT TAPE 7,406,P(1),BET(1),DEL(1),SRAN(1),AMD(1)
      WRITE OUTPUT TAPE 15,401
      WRITE OUTPUT TAPE 15,402,AZ,ARTHO(1)
      WRITE OUTPUT TAPE 15,407,AE(1),RANGE,RANGM,SZZUS(1)
      WRITE OUTPUT TAPE 15,404,H,B,D,E,F
      WRITE OUTPUT TAPE 15,405,G,ZAM,Q,R
      WRITE OUTPUT TAPE 15,406,P(1),BET(1),DEL(1),SRAN(1),AMD(1)
      WRITE OUTPUT TAPE 15,403
5 CALL DPADD(AE,P,AEP)
  CALL DPDIV(AE,AEP,DA)
  C2(1)=2.0
  C2(2)=0.0
  RED=-30.
  KJ=-1
  C3(1)=1.0
  C3(2)=0.0
  GAY=0.0
  RAN(1)=SRAN(1)
  RAN(2)=SRAN(2)
  DRE=0.0
  R13P=0.0
  KC=0
  SBCC=0.0
  NOS=-1
  KLIEM=-1
  SSI=-1.0
8 WHY=RWBY

```

```

YHW=1.0
C1(1)=4.0
C1(2)=0.0
CALL DPMPY(C1,BET,ST1)
CALL DPADD(ST1,SZUS,ZZUS)
GO TO 30
29 CALL DPSUB(ZZUS,BET,ZZUS)
30 IF(WHY-YHW)31,31,32
31 CALL DPSUB(RAN,DEL,RAN)
GO TO 625
32 CALL DPADD(RAN,DEL,RAN)
625 IF(RAN(1)-STRAN)156,33,33
33 IF(RAN(1)-RANGM)34,34,41
34 IF (RAN-ARTHO) 35,10,36
10 IF (RAN(2)-ARTHO(2)) 35,35,36
35 Z2M(1)=0.0
Z2M(2)=0.0
GO TO 37
36 CALL DPSUB(RAN,ARTHO,BB)
CALL DPMPY(AE,AE,ST1)
CALL DPMPY(BB,BB,ST2)
CALL DPADD(ST1,ST2,ST3)
CALL DPROOT(ST3,ST1)
CALL DPSUB(ST1,AE,Z2M)
37 BCC=PRS
FRED= ZZUS(1)
FRED(2)=ZZUS(2)
NRC=-200
AKA=0.0
SENSE LIGHT 1
GO TO 39
38 SENSE LIGHT 0
39 CALL DPADD(ZZUS,BET,ZZUS)
GO TO 540
40 CALL DPSUB(ZZUS,BET,ZZUS)
540 IF (ZZUS-Z2M) 460,541,42
541 IF (ZZUS(2)-Z2M(2)) 460,42,42
460 CONTINUE
462 IF(SENSE SWITCH 3)463,464
463 PRINT 614,Z2M(1),ZZUS(1),RAN(1)
614 FORMAT(5H0Z2M=E17.9,6H ZZUS=E17.9,5H RAN=E17.9)
464 WRITE OUTPUT TAPE 7,614,Z2M(1),ZZUS(1),RAN(1)
GO TO 39
41 IF(SENSE SWITCH 1)465,466
465 PRINT 615,RAN(1)
615 FORMAT(1H1/5H RAN=E17.9,35H RAN IS GREATER THAN MAX RANGE STOP)
466 WRITE OUTPUT TAPE 7,615,RAN(1)
GO TO 156
42 IF(ZZUS(1)-ZAM)543,543,43
43 IF(SENSE SWITCH 1)467,468
467 PRINT 616,ZZUS(1)
616 FORMAT(1H1/6H ZZUS=E17.9,37H ZZUS IS GREATER THAN MAX TOT HT STOP)
468 WRITE OUTPUT TAPE 7,616,ZZUS(1)

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GO TO 750
543 CONTINUE
C 1-COS(THETA1)=(R(2)**2-(Z(1)-Z(2))**2)/2(AE+Z(1))(AE+Z(2))-VSANG
118 CALL DPSUB(P,ZZUS,ST1)
CALL DMPY(ST1,ST1,ST1)
CALL DMPY(RAN,RAN,ST2)
CALL DPSUB(ST2,ST1,ST1)
CALL DPADD(AE,ZZUS,AEZ)
CALL DMPY(AEZ,C2,ST2)
CALL DMPY(ST2,AEF,ST2)
CALL DFDIV(ST1,ST2,VSANG)
C COS(THETA(1))=1-VSANG-COANG
CALL DPSUB(C3,VSANG,COANG)
C SIN(THETA(1))=SQRTF(1-COANG**2)=SQRTF(1-COANG)(1+COANG)
CALL DPADD(C3,COANG,ST1)
CALL DMPY(VSANG,ST1,ST1)
CALL DPROOT(ST1,SIANG)
ANG=ATANF(SIANG/COANG)
C DB=AE/(AE+ZZUS)
CALL DFDIV(AE,AEZ,DB)
IF(SENSE SWITCH 6)477,478
478 IF(SENSE SWITCH 5)479,480
479 PRINT 619,RAN(1),ZZUS(1),ANG(1),ANG2(1)
619 FORMAT(5HORAN=E17.9,6H ZZUS=E17.9,5H ANG=E17.9,6H ANG2=E17.9)
480 WRITE OUTPUT TAPE 7,619,RAN(1),ZZUS(1),ANG(1),ANG2(1)
477 SENSE LIGHT 3
IND=-1
IF(SSI)830,836,836
836 ASSIGN 832 TO K
GO TO 130
830 GEA=(P(1)+ZZUS(1))/RAN(1)
GEB=ATANF(GEA)
GEC=1.57079632-GEB
GED=ZZUS(1)/GEC
C SIN(THETA)=THETA
SI(1)=GED/AE(1)
SI(2)=0.0
831 ASSIGN 832 TO K
GO TO 130
832 ASSIGN 147 TO K
IF(SENSE LIGHT 3)835,845
835 FRY(1)=TRY(1)
FRY(2)=TRY(2)
IF(SENSE SWITCH 6)901,902
902 IF(SENSE SWITCH 5)903,904
903 PRINT 632,TAG1(1),TAG2(1),FRY(1)
632 FORMAT(52HO*****FLAT EARTH APPROXIMATION*****6H
XTAG1=E17.9,6H TAG2=E17.9,5H FRY=E17.9)
PRINT 631,SI(1),SII(1),CO(1),Z(1),ZB(1)
904 WRITE OUTPUT TAPE 7,632,TAG1(1),TAG2(1),FRY(1)
WRITE OUTPUT TAPE 7,631,SI(1),SII(1),CO(1),Z(1),ZB(1)
901 IF(TAG1)920,920,921
921 IF(TAG2)920,920,923

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C                                     IF TANGENT NEGATIVE
920 IF(IND)927,925,925
C SIN(RAY)=SIN(ANG)COS(ANG-RAY)-SIN(ANG-RAY)COS(ANG)
927 CALL DMPY(SIANG,AE,ST1)
CALL DMPY(COANG,ARTHO,ST2)
CALL DPSUB(ST1,ST2,ST1)
C IF ARP LINE OF SIGHT OVERSHOOT OF TARGET*****
IF(ST1)940,940,941
940 SI=0.0
SI(2)=0.0
SI1=0.0
SI1(1)=0.0
CO=1.0
CO(2)=0.0
GO TO 942
941 CALL DPDIV(ST1, AEP, SI)
C COS(RAY)=COS(ANG)COS(ANG-RAY)+SIN(ANG)SIN(ANG-RAY)
CALL DMPY(COANG,AE,ST1)
CALL DMPY(SIANG,ARTHO,ST2)
CALL DPADD(ST1,ST2,ST1)
CALL DPDIV(ST1,AEP,CO)
C AVA=SIN(Y)/3.
C =((SQRT(ZZUS(2.*AE+ZZUS)))*CO-SI*AE)/(AE+ZZUS)*3.
C IF TGT LINE OF SIGHT OVERSHOOT OF ARP*****
942 CALL DPADD(AEZ,AE,ST1)
CALL DMPY(ST1,ZZUS,ST1)
CALL DPROOT(ST1,ST1)
CALL DPDIV(ST1,AEZ,ST1)
IF(ST1-SIANG)945,943,944
943 IF(ST1(2)-SIANG(2))945,944,944
944 ST1=SIANG
ST1(2)= SIANG(2)
ST2=COANG
ST2(2)=COANG(2)
GO TO 946
945 CALL DPDIV(AE,AEZ,ST2)
946 CALL DMPY(ST1,CO,ST1)
CALL DMPY(ST2,SI,ST2)
CALL DPSUB(ST1,ST2,ST1)
C SINE OF THE DIFFERENCE DIVIDED BY 3.0*****
AVA=ST1/3.
AVA(2)=0.0
SENSE LIGHT 3
IF(AVA)925,925,928
C SI=SI+AVA
928 CALL DPADD(SI,AVA,SI)
IND=IND+1
GO TO 831
925 PRINT 930,RAN(1),ZZUS(1)
930 FORMAT(36HOUNABLE TO FIND REFLECTION POINT RAN=E17.9,6H ZZUS=E17.9)
933 WRITE OUTPUT TAPE 7,930,RAN(1),ZZUS(1)
GO TO 38
923 SI1(1)=SI(1)

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      SI1(2)=SI(2)
      IF(IND)929,928,929
929 IF(SSI)833,834,834
834 SI(1)=(SI1(1)/1000.)+SI1(1)
      SI(2)=(SI1(2)/1000.)+SI1(2)
      GO TO 831
833 GEF=AE*((1.0/P)+(1.0/ZZUS))
      SI(1)=(TRY(1)/(1000.0*GEF))+SI1(1)
      SI(2)=0.0
      GO TO 831
845 IF(SENSE SWITCH 6)846,907
907 IF(SENSE SWITCH 5)908,909
908 PRINT 630,TAG1(1),TAG2(1),FRY(1)
630 FORMAT(52H0*****INCREMENTED FLAT EARTH APPROXIMATION*****6H
      XTAG1=E17.9,6H TAG2=E17.9,5H FRY=E17.9)
      PRINT 631,SI(1),SI1(1),CO(1),Z(1),ZB(1)
631 FORMAT(4H SI=E17.9,5H SI1=E17.9,4H CO=E17.9,3H Z=E17.9,4H ZB=E17.9 X)
909 WRITE OUTPUT TAPE 7,630,TAG1(1),TAG2(1),FRY(1)
      WRITE OUTPUT TAPE 7,631,SI(1),SI1(1),CO(1),Z(1),ZB(1)
846 IF(TAG1)920,920,924
924 IF(TAG2)920,920,148
C                                CO=SQRT(1-SI**2)
130 CALL DMPY(SI,SI,ST1)
      CALL DPSUB(C3,ST1,ST1)
      CALL DPROOT(ST1,CO)
C                                ZB=SIANG*CO-SI*COANG
      CALL DMPY(SIANG,CO,ST1)
      CALL DMPY(SI,COANG,ST2)
      CALL DPSUB(ST1,ST2,ZB)
C                                Z=CO*COANG+SI*SIANG
      CALL DMPY(CO,COANG,ST1)
      CALL DMPY(SI,SIANG,ST2)
      CALL DPADD(ST1,ST2,Z)
C                                TAG1=ZB/(Z-(AE/(AE+P)))
      CALL DPSUB(Z,DA,ST1)
      CALL DPDIV(ZB,ST1,TAG1)
C                                TAG2=SI/(CO-(AE/(AE+ZZUS)))
      CALL DPSUB(CO,DB,ST1)
      CALL DPDIV(SI,ST1,TAG2)
      NRC=NRC+1
      IF (NRC)145,145,163
145 CALL DPSUB(TAG1,TAG2,TRY)
      GO TO K,(147,832)
147 CONTINUE
C                                AVA=TRY*((SI-SI1)/(FRY-TRY))
C
C      CALCULATE NEW SI
148 CALL DPSUB(SI,SI1,CE1M2)
      CALL DPSUB(FRY,TRY,DL1M2)
      CALL DPDIV(CE1M2,DL1M2,ST1)
      CALL DMPY(TRY,ST1,AVA)
      ST1=AMD*SI
      SI1=SI
      SI1(2)=SI(2)

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      FRY=TRY
      FRY(2)=TRY(2)
      CALL DPADD(AVA,SI,SI)
481 IF(SENSE SWITCH 6)495,482
482 IF(SENSE SWITCH 5)483,484
483 PRINT 620,SI(1),SI(1),CO(1),Z(1),ZB(1)
620 FORMAT(4HOSI=E17.9,5H SI=E17.9,4H CO=E17.9,3H Z=E17.9,4H ZB=E17.9 X)
      PRINT 621,RAY1,AVA(1),FRY(1),TAG1(1)
621 FORMAT(6H RAY1=E17.9,5H AVA=E17.9,5H FRY=E17.9,6H TAG1=E17.9)
484 WRITE OUTPUT TAPE 7,620,SI(1),SI(1),CO(1),Z(1),ZB(1)
      WRITE OUTPUT TAPE 7,621,RAY1,AVA(1),FRY(1),TAG1(1)
      GO TO 495
495 IF(TAG1)920,920,936
936 IF(TAG2)920,920,937
937 IF (ABSF (AVA) - ST1) 151,151,130
C   COS(APH01)=1.0/SQRT(1.0+TAG1**2.)=ST1*****
151 CALL DPMFY(TAG1,TAG1,ST1)
      CALL DPADD(ST1,C3,ST2)
      CALL DPROOT(ST2,ST1)
      CALL DPDIV(C3,ST1,ST1)
C   COS(APH02)=1.0/SQRT(1.0+TAG2**2.)=ST3*****
      CALL DPMFY(TAG2,TAG2,ST2)
      CALL DPADD(ST2,C3,ST2)
      CALL DPROOT(ST2,ST2)
      CALL DPDIV(C3,ST2,ST3)
C   RANG1=AE(((Z-1.0)+(P*Z)/AE)/COS(APH01))*****
      CALL DPMFY(Z,P,ST2)
      CALL DPDIV(ST2,AE,ST2)
      CALL DPSUB(Z,C3,ST4)
      CALL DPADD(ST4,ST2,ST2)
      CALL DPDIV(ST2,ST1,ST2)
      CALL DPMFY(ST2,AE,RANG1)
C   RANG3=AE(((COS(RAY)-1.0)+(Z2*COS(RAY))/AE)/COS(APH02))*****
      CALL DPMFY(Z2US,CO,ST2)
      CALL DPDIV(ST2,AE,ST2)
      CALL DPSUB(CO,C3,ST4)
      CALL DPADD(ST2,ST4,ST2)
      CALL DPDIV(ST2,ST3,ST2)
      CALL DPMFY(ST2,AE,RANG3)
C   RANG13=RANG1+RANG3-RAN*****
      CALL DPADD(RANG1,RANG3,ST4)
      CALL DPSUB(ST4,RAN,RANG13)
      APH0=ATANF((TAG1+TAG2)/2.0)
      APH01=ATANF(TAG1)
      APH02=ATANF(TAG2)
      PHS=RANG13/B
      IF(SENSE SWITCH 2)485,486
486 IF(SENSE SWITCH 5)487,488
487 PRINT 622,TAG1(1),APH01(1),TAG2(1),APH02(1)
622 FORMAT(6HOTAG1=E17.9,7H APH01=E17.9,21H          TAG2=E17.9,
      X7H APH02=E17.9)
488 WRITE OUTPUT TAPE 7,622,TAG1(1),APH01(1),TAG2(1),APH02(1)
485 ATA=3.14159265-APH0

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SSI=+1.0
SI2=1.57079632-APHO
RAY1=ATANF(SI1/CO)
SI3=3.14159265-ATA-RAY1
SI4=3.14159265-ATA-(ANG(1)-RAY1)
SA=(RANG1(1)+RANG3(1)+RAN(1))/2.0
SB=(SA-RAN(1))*(SA-RANG1(1))
SC=SA*(SA-RANG3(1))
SQAC=SQRTF(SB/SC)
TATA2--(1.57079632-SI4)
TATAL=2.0*(ATANF(SQAC))-ABSF(TATA2)
BR6=60.0*B*R
C2APH0=COSF(SI2)*COSF(SI2)
GA=Q-C2APH0
GB=-ABSF(BR6)
GC=(GA*GA)+(GB*GB)
GD=SQRTF(GC)
BETA2=ATANF(GB/GA)
BETA1=BETA2/2.0
GDR=SQRTF(GD)
CBETA1=COSF(BETA1)
SBETA1=SINF(BETA1)
GE=CBETA1*GDR
GF=SBETA1*GDR
DZ=0.5
C CURVED AND FLAT EARTH DIFF=(RAN**2)/2.0*SAE RANGE=RAN*COS(TATAL)
SUZ2US=(RAN**2)/(2.0*SAE)
PZ2US=Z2US-SUZ2US
IF(GAM-DZ)152,152,153
152 GG=Q*SINF(SI2)
GH=-BR6*SINF(SI2)
GI=GG-GE
GJ=GH-GF
GK=GG+GE
GL=GH+GF
GM=(GI*GI)+(GJ*GJ)
GN=SQRTF(GM)
BETA3=ATANF(GJ/GI)
GO=(GK*GK)+(GL*GL)
GP=SQRTF(GO)
BETA4=ATANF(GL/GK)
GQ=GN/GP
GR=BETA3-BETA4
GMVR=GMDO*(GQ*COSF(GR))
GMVI=GQ*SINF(GR)
GO TO 155
153 GS=SINF(SI2)-GE
GT=-GF
GU=SINF(SI2)+GE
GV=(GS*GS)+(GT*GT)
GW=SQRTF(GV)
BETA5=ATANF(GT/GS)
BETA6=ATANF(GF/GU)

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GX=(GU*GU)+(GF*GF)
GY=SQRTF(GX)
GZ=GW/GY
HA=BETA5-BETA6
GMHR=GMDO*(GZ*COSF(HA))
GMHI=GZ*SINF(HA)
155 HB=SINF(APHO)*COSF(APHO)
HC=SQRTF(HB)
HD=RANG1(1)+RANG3(1)
HE=AE(1)*HD*HC
HF=(AE(1)+Z2US(1))*SINF(ANG(1))
HG=(AE(1)+P(1))*HF
HI=(AE(1)+Z2US(1))*RANG1(1)*COSF(SI3)
HJ=(AE(1)+P(1))*RANG3(1)*COSF(SI4)
HK=HJ+HI
HL=HK*HG
HM=SQRTF(HL)
DE=HE/HM
DELT=(RANG13(1)/B)*6.28318530
DELTM=MODF(DELT,6.28318530)
CSD=COSF(DELTM)
SND=SINF(DELTM)
DZ=0.50
IF(GAM-DZ)349,349,350
349 GMR=GMVR
GMI=GMVI
GO TO 352
350 GMR=GMHR
GMI=GMHI
352 PB=GMR*CSD
PC=GMR*SND
PD=GMI*CSD
PE=GMI*SND
PF=PB+PE
PG=PD-PC
AITA2=((TATA2*57.2957795)-90.0)
AITA1=((TATA1*57.2957795)-90.0)
J=XINTF(AITA2)
ALOT2=A(J)
J=J+1
FLOA2=FLOATF(J)
ALOT21=A(J)
J=XINTF(AITA1)
ALOT1=A(J)
J=J+1
FLOA1=FLOATF(J)
ALOT11=A(J)
ADDA2=(ALOT21-ALOT2)*(FLOA2-AITA2)/1.0
ADDA1=(ALOT11-ALOT1)*(FLOA1-AITA1)/1.0
FOT1=ALOT11-ADDA1
FOT2=ALOT21-ADDA2
PR=FOT2*DE*PF
PI=FOT2*DE*PG

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FOE1=FOT1+FR
FOE2=PI
FOE=(FOE1*FOE1)+(FOE2*FOE2)
FELD1=SQRT(FOE)
AYK2=((0.5)*RANGM)/RAN(1)
SPWR=FWR1
PAY1=AYK2*FELD1
FWR1=PAY1*PAY1*PAY1*PAY1
IF(TA)115,115,44
44 IF(WHY-YHW)68,68,69
69 IF(KC-2)68,68,70
70 IF(FWR-1.0)71,115,115
71 IF(SPWR-FWR1)68,68,175
68 IF(BCC-(PHS+RBCC))175,175,115
175 KJ=0
IF(WHY-YHW)47,47,46
46 IF(RANG13-R13P)160,160,48
48 R13P=RANG13
GO TO 75
47 KC=0
SBCC=SBCC+1.0
GO TO 50
115 IF(WHY-YHW)51,51,76
50 WHY=2.0
CALL DPADD(RAN,DEL,RAN)
Z2US(1)=FRED+BET-(FZ2US*2.)
IF(SENSE SWITCH 1)475,474
474 IF(SENSE SWITCH 5)29,476
475 PRINT 618,RAN(1),Z2US(1),FRED(1),WHY,NRC
618 FORMAT(31HOLOBE CHANGE EXIT 4 RAN=E17.9,6H Z2US=E17.9,6H FR
XED=E17.9,5H WHY=E10.3,5H NRC=13)
476 WRITE OUTPUT TAPE 7,618,RAN(1),Z2US(1),FRED(1),WHY,NRC
GO TO 29
51 IF(FWR1-1.0)60,61,61
60 IF(BET(1)-AKA)64,444,444
444 KC=KC+1
IF(SENSE SWITCH 4)101,102
102 IF(SENSE SWITCH 5)103,104
103 PRINT 110,FWR1,RAN(1),Z2US(1),PHS,DE,GMR,GMI,FOT1,FOT2,NRC,IND
110 FORMAT(17HOEXIT 10 10 FWR1=E9.2,5H RAN=-3PF8.3,4H Z2=OFF10.3,5H PH
XS=F8.2,4H DE=F4.2,4H GM=2F6.3,5H FOT=2F5.2,5H NRC=13,5H IND=12)
104 WRITE OUTPUT TAPE 7,110,FWR1,RAN(1),Z2US(1),PHS,DE,GMR,GMI,FOT1,FO
XT2,NRC,IND
GO TO 40
101 IF(SENSE SWITCH 2)40,447
447 WRITE OUTPUT TAPE 7,610
610 FORMAT(11HOEXIT 10 10)
IF(SENSE SWITCH 3)446,449
446 PT=1.0
PRINT 610
GO TO 448
64 TA=1.0
IF(NOS)84,84,85

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84 SBCC=INTF(PBS)
   NOS=1
85 NO=NO+1
   KC=0
   IF(SENSE SWITCH 1)450,451
450 PRINT 606,NO,PWR1,RAN(1),Z2US(1),PZ2US,PBS
   DRE=DRE+1.0
   IF(DRE-ERD)451,860,860
860 PRINT 607
180 DRE=0.0
451 WRITE OUTPUT TAPE 15,410,NO,PWR1,RAN(1),Z2US(1),PZ2US,PBS
   IF(SENSE SWITCH 2)16,17
16 IF(SENSE SWITCH 4)18,19
18 IF(PRV)19,22,22
17 WRITE OUTPUT TAPE 7,606,NO,PWR1,RAN(1),Z2US(1),PZ2US
   WRITE OUTPUT TAPE 7,602,RANG1(1),RANG3(1),RANG13(1),PBS,NRC,IND
   WRITE OUTPUT TAPE 7,603,FOT1,TATA1,FOT2,TATA2,FLOA1,FLOA2
   WRITE OUTPUT TAPE 7,604,DE,GMR,GMI,SI2,SI3,SI4
   GO TO 22
19 WRITE OUTPUT TAPE 7,23,NO,TAG1(1),APH01(1),FOT1,FOT2,DE,GMR,GMI,NR
   XC,IND
23 FORMAT(4H NO=16,6H TAG1=E10.3,7H APH01=E10.3,6H FOT1=E10.3,6H FOT2
   X=E10.3,4H DE=F4.2,5H GMR=F6.4,5H GMI=F6.4,5H NRC=13,5H IND=12)
22 IF(KJ)171,172,173
171 SFZ2US=Z2US
   KJ=1
   Z2US=Z2US+SLZ2US
167 RED=RED+1.0
   IF(RED-ERD)31,611,611
173 FZ2US=Z2US-SFZ2US
   SFZ2US=Z2US
   Z2US=Z2US+FZ2US
   GO TO 167
172 SFZ2US=Z2US
   Z2US=Z2US-FZ2US
   KJ=1
   GO TO 167
611 WRITE OUTPUT TAPE 15,607
   WRITE OUTPUT TAPE 15,403
403 FORMAT(12H PRINT NO.,20H RELATIVE POWER,17H RANGE IN K
   XM,22H HEIGHT IN METERS,21H PLOTTING HEIGHT,24H MODU
   XLO PHASE SHIFT/1HO)
   RED=-30.0
   GO TO 31
61 AKA=1.0+BET(1)
65 IF(SENSE LIGHT 1)66,67
66 FRED=Z2US
67 KC=KC+1
   IF(SENSE SWITCH 4)27,28
28 IF(SENSE SWITCH 5)52,53
52 PRINT 58,PWR1,RAN(1),Z2US(1),PBS,DE,GMR,GMI,FOT1,FOT2,NRC,IND
58 FORMAT(17HOEXIT 11 11 PWR1=E9.2,5H RAN=-3F8.3,4H Z2=OPT10.3,5H PH
   XS=F8.2,4H DE=F4.2,4H GM=2F6.3,5H FOT=2F5.2,5H NRC=13,5H IND=12)

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53 WRITE OUTPUT TAPE 7,58,FWRL,RAN(1),Z2US(1),PHS,DE,GMR,GMI,FOT1,FOT
  X2,NRC,IND
  GO TO 38
27 IF(SENSE SWITCH 2)38,455
455 WRITE OUTPUT TAPE 7,612
612 FORMAT(11HOEXIT 11 11)
  IF(SENSE SWITCH 3)454,457
454 PT=1.0
  PRINT 612
  GO TO 456
75 WHY=0.0
  CALL DPSUB(RAN,DEL,RAN)
  Z2USL=Z2US(1)
  Z2US=FRED+BET-(FZ2US*2.)
  IF(SENSE SWITCH 1)472,473
472 PRINT 617,RAN(1),Z2USL,FRED(1),WHY,NRC,Z2US(1)
617 FORMAT(31HOLOBE CHANGE EXIT 14 RAN=E12.4,7H Z2USL=E12.4,6H F
  XRED=E12.4,5H WHY=E9.2,5H NRC=13,6H Z2US=E12.4)
473 WRITE OUTPUT TAPE 7,617,RAN(1),Z2USL,FRED(1),WHY,NRC,Z2US(1)
  GO TO 30
76 IF(FWRL-1.0)78,79,79
78 IF(BET(1)-AKA)81,92,92
79 IF(KLIEM)87,87,86
87 KC=0
  IF(Z2US(1)+(-BET-AIZ-Z2M))82,80,80
80 AKA=1.0+BET(1)
  KC=KC+1
  IF(SENSE SWITCH 4)105,106
106 IF(SENSE SWITCH 5)107,108
107 PRINT 111,FWRL,RAN(1),Z2US(1),PHS,DE,GMR,GMI,FOT1,FOT2,NRC,IND
111 FORMAT(14HOEXIT 10 FWRL=E9.2,5H RAN=-3PF8.3,4H Z2=OFF10.3,5H PHS=F
  X8.2,4H DE=F4.2,4H GM=2F6.3,5H FOT=2F5.2,5H NRC=13,5H IND=12)
108 WRITE OUTPUT TAPE 7,111,FWRL,RAN(1),Z2US(1),PHS,DE,GMR,GMI,FOT1,FO
  XT2,NRC,IND
  GO TO 40
105 IF(SENSE SWITCH 2)40,421
421 IF(SENSE SWITCH 3)422,423
422 PRINT 601
  PT=-1.0
601 FORMAT(8HOEXIT 10)
448 PRINT 605,FWRL,RAN(1),Z2US(1)
605 FORMAT(6H FWRL=E17.9,6H RAN=E17.9,6H Z2US=E17.9)
  PRINT 602,RANG1(1),RANG3(1),RANG13(1),PHS,NRC,IND
602 FORMAT(7H RANG1=E17.9,8H RANG3=E17.9,9H RANG123=E17.9,5H PHS=F8.2
  X,5H NRC=13,5H IND=12)
  PRINT 603,FOT1,TATA1,FOT2,TATA2,FLOA1,FLOA2
603 FORMAT(6H FOT1=E12.5,7H TATA1=E17.9,6H FOT2=E12.5,7H TATA2=E17.9.6
  XH J(1)=E10.3,6H J(2)=E10.3)
  PRINT 604,DE,GMR,GMI,SI2,SI3,SI4
604 FORMAT(4H DE=E17.9,5H GMR=E12.5,5H GMI=E12.5,5H SI2=E16.8,5H SI3=E
  X16.8,5H SI4=E16.8)
420 IF(PT)423,423,449
423 WRITE OUTPUT TAPE 7,601

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449 WRITE OUTPUT TAPE 7,605,FWRL,RAN(1),ZZUS(1)
    WRITE OUTPUT TAPE 7,602,RANG1(1),RANG3(1),RANG13(1),PBS,MRC,IND
    WRITE OUTPUT TAPE 7,603,FOT1,TATA1,FOT2,TATA2,FLOA1,FLOA2
    WRITE OUTPUT TAPE 7,604,DE,GMR,GMI,SI2,SI3,SI4
    GO TO 40
81 IF(SBCC-(PBS+0.5))83,86,86
86 ZZUS=ZZUS+BET
    KLIEM=1
    GO TO 540
83 TA=1.0
    NO=NO+1
    KLIEM=-1
    KC=0
82 IF(SENSE SWITCH 1)430,431
430 PRINT 606,NO,FWRL,RAN(1),ZZUS(1),PZZUS,PBS
606 FORMAT(1H016,16H RELATIVE POWER=E17.9.8H RANGE=-3PF8.2,11H KM HEI
    XGET=OFF8.2,9H M PLOT=F8.2,15H M PHASE SHIFT=F13.7)
    DRE=DRE+1.0
    IF(DRE-ERD)431,459,459
459 PRINT 607
607 FORMAT(1H1)
181 DRE=0.0
431 WRITE OUTPUT TAPE 15,410,NO,FWRL,RAN(1),ZZUS(1),PZZUS,PBS
410 FORMAT(1I0,E24.9,-3PF13.2,OFF20.2,F21.2,F25.7)
    IF(SENSE SWITCH 2)20,21
20 IF(SENSE SWITCH 4)24,25
24 IF(PRV)25,26,26
21 WRITE OUTPUT TAPE 7,606,NO,FWRL,RAN(1),ZZUS(1),PZZUS
    WRITE OUTPUT TAPE 7,602,RANG1(1),RANG3(1),RANG13(1),PBS,MRC,IND
    WRITE OUTPUT TAPE 7,603,FOT1,TATA1,FOT2,TATA2,FLOA1,FLOA2
    WRITE OUTPUT TAPE 7,604,DE,GMR,GMI,SI2,SI3,SI4
    GO TO 26
25 WRITE OUTPUT TAPE 7,23,NO,TAG1(1),APH01(1),FOT1,FOT2,DE,GMR,GMI,NR
    XC,IND
26 IF(KJ)168,169,170
168 SFZZUS=ZZUS
    KJ=1
166 RED=RED+1.0
    IF(RED-ERD)32,608,608
170 FZZUS=ZZUS-SFZZUS
    SFZZUS=ZZUS
    ZZUS=ZZUS+FZZUS
    GO TO 166
169 SFZZUS=ZZUS
    ZZUS=ZZUS-FZZUS
    KJ=1
    GO TO 166
608 WRITE OUTPUT TAPE 15,607
    WRITE OUTPUT TAPE 15,403
    RED=-30.0
    GO TO 32
92 KLIEM=-1
    IF(SENSE LIGHT 1)93,94

```

```

93 FRED=Z2US
94 KC-KC+1
   IF(SENSE SWITCH 4)54,55
55 IF(SENSE SWITCH 5)56,57
56 PRINT 59,FWRL,RAN(1),Z2US(1),PES,DE,GMR,GMI,FOT1,FOT2,NRC,IND
59 FORMAT(14HOEXIT 11 FWRL=E9.2,5H RAN=-3PF8.3,4H Z2=OPF10.3,5H PES=F
X8.2,4H DE=F4.2,4H GM=2F6.3,5H FOT=2F5.2,5H NRC=13,5H IND=12)
57 WRITE OUTPUT TAPE 7,59,FWRL,RAN(1),Z2US(1),PES,DE,GMR,GMI,FOT1,FOT
X2,NRC,IND
   GO TO 38
54 IF(SENSE SWITCH 2)38,458
458 IF(SENSE SWITCH 3)432,433
432 PRINT 609
   PT=-1.0
609 FORMAT(8HOEXIT 11)
456 PRINT 605,FWRL,RAN(1),Z2US(1)
   PRINT 602,RANG1(1),RANG3(1),RANG13(1),PES,NRC,IND
   PRINT 603,FOT1,TATA1,FOT2,TATA2,FLOA1,FLOA2
   PRINT 604,DE,GMR,GMI,SI2,SI3,SI4
   IF(PT)433,433,457
433 WRITE OUTPUT TAPE 7,609
457 WRITE OUTPUT TAPE 7,605,FWRL,RAN(1),Z2US(1)
   WRITE OUTPUT TAPE 7,602,RANG1(1),RANG3(1),RANG13(1),PES,NRC,IND
   WRITE OUTPUT TAPE 7,603,FOT1,TATA1,FOT2,TATA2,FLOA1,FLOA2
   WRITE OUTPUT TAPE 7,604,DE,GMR,GMI,SI2,SI3,SI4
   GO TO 38
160 PRINT 184
   WRITE OUTPUT TAPE 7,184
184 FORMAT (1H1/13H LOWER RANG13)
   ASSIGN 75 TO K1
161 PRINT 183,RAN(1),Z2US(1),RANG13(1),R13P
   WRITE OUTPUT TAPE 7,183,RAN(1),Z2US(1),RANG13(1),R13P
183 FORMAT (5H RAN= E17.9,6H Z2US= E17.9,8H RANG13= E17.9,6H R13P=E1
17.9 /1H0/53HO*****SIMULATED PROGRAM STOP*****
2/40HO PROGRAMED PDUMP NOW GOING ON COMMUNITY/1H0/1H0)
   L=XLOC(SLOC)-99
   CALL PDUMP(SLOC(L),SLOC(1),3)
   PRINT 185
   WRITE OUTPUT TAPE 7,185
   END FILE 7
   END FILE 15
185 FORMAT(91HOPROGRAMED PDUMP COMPLETED*****EXIT CALLED*****
LEND OF PROGRAM*****FRED LOWELL)
   CALL EXIT
162 PAUSE
   GO TO K1,(75,145)
163 PRINT 182
   WRITE OUTPUT TAPE 7,182
182 FORMAT (1H1/26H 200 ANGLES WITHOUT OUTPUT)
   NRC=-200
   ASSIGN 145 TO K1
156 GO TO 161
624 FORMAT(1H1/29H END OF PROGRAM F LOWELL RAN=E17.9,6H Z2US=E17.9)

```

```
750 PRINT 624,RAN(1),Z2US(1)
    END FILE 7
    END FILE 15
    CALL EXIT
    END(1,1,0,0,1,0,0,0,0,0,0,0,0,0)
```

|
9

```
750 PRINT 624,RAN(1),Z2US(1)
END FILE 7
END FILE 15
CALL EXIT
END(1,1,0,0,1,0,0,0,0,0,0,0,0,0)
```

CHAPTER VII

CONCLUSION

A method for a rigorous theoretical analysis of the pattern of coverage of transmitting equipment in the earth's atmosphere requires equations which are cumbersome to solve. The formulation of ΔR by the transcendental approach of the author yields results which may be sought to a high degree of accuracy with a minimum of work. The overall concepts of reflection, divergence, and the index of refraction of the atmosphere may be taken into account. The analysis has been shown to be only feasibly solved by a digital computer.

Further work should be directed to one of two areas. The first is that of obtaining an index of refraction other than that which yields the $4/3$ earth. It appears that no constant value can be assumed throughout the region of entire space. Any variances from the standard $4/3$ earth noticeably causes deviation in any pattern of coverage. The second is through the utilization of a normalizing procedure of the equations. This would noticeably reduce the work required in performing the calculations.

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APPENDICES

APPENDIX A

The following two graphs should supplement Chapter III and Chapter V of this report. Figure A-1 shows the dependence of ΔR in μ seconds as a function of the range R_2 . The transmitter and target heights are maintained at constant values. Figure A-1 was obtained by the exact equations.

Figure A-2 shows the dependence of ΔR , as a function of R_2 , on the method used for calculation. The exact and cubic formulations of ΔR are compared. All calculations of this appendix were made by N.I. Durlach, A.M. Carpenter, and M.A. Herlin of Group 45 at Lincoln Laboratory.

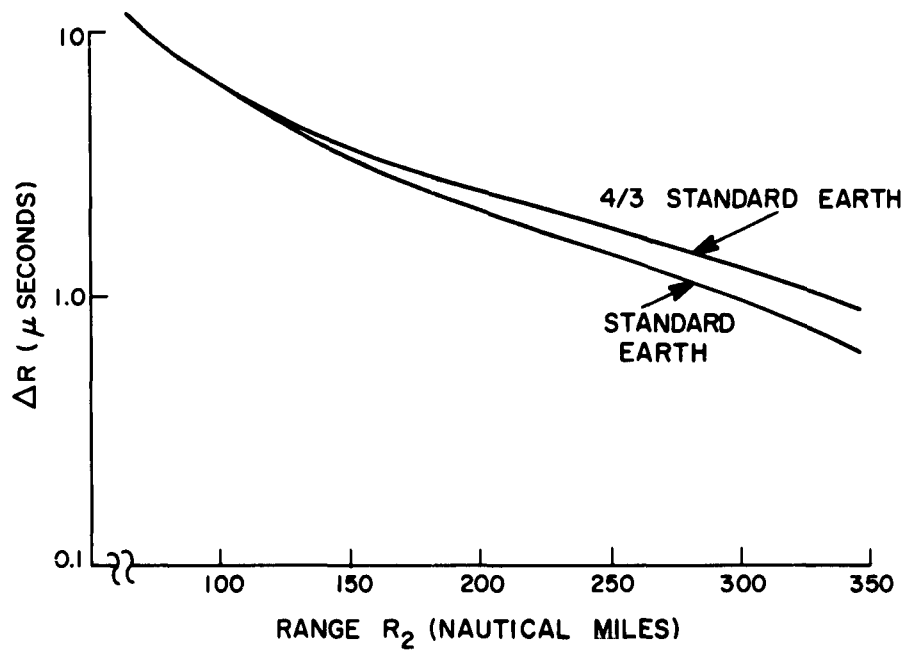


Figure A-1

ΔR in μ seconds as a function of R_2 as calculated by the exact expressions for a standard and 4/3 earth. The height of the transmitter is 40,000 feet and the height of the receiver or target is 60,000 feet.

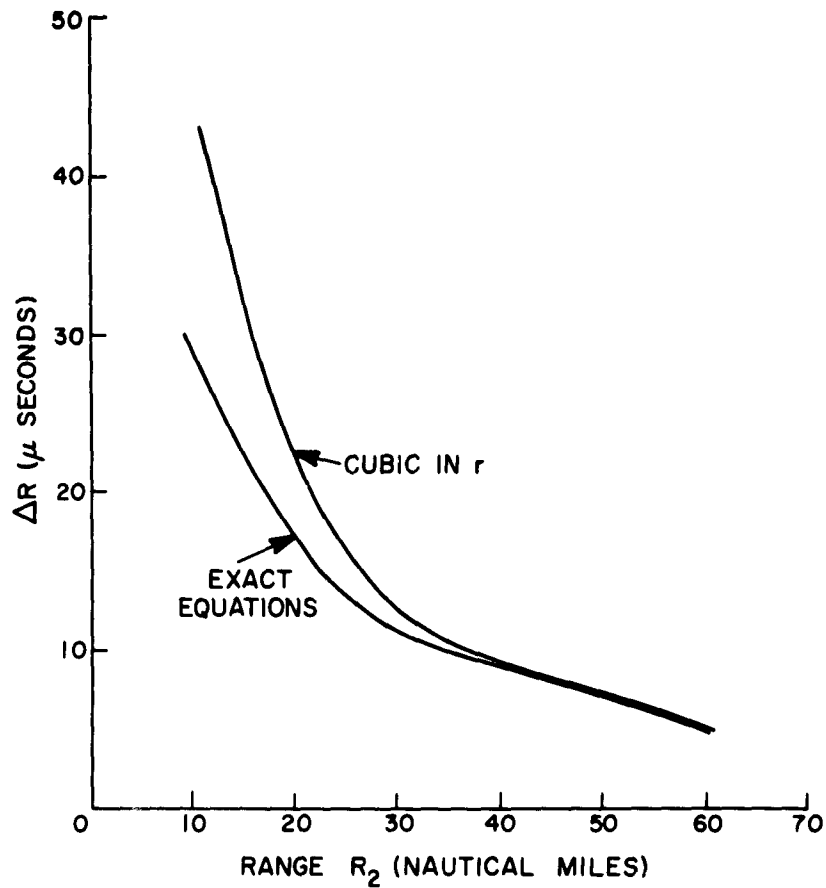


Figure A-2

ΔR in μ seconds as a function of R_2 as calculated for a $4/3$ earth by the exact equations and the equations represented in the section of the cubic r . The height of the transmitter is 20,000 feet and the height of the target or receiver is 60,000 feet.

APPENDIX B

The following figure and tables were obtained by assuming typical radar parameters as follow. The maximum gain of the antenna is 251, the wave length of the propagated waves is 0.705 meters, the peak power output of the transmitter is 2×10^6 watts, the minimum useful signal is 4×10^{-15} watts, the target at Z_2 has an effective cross-sectional area of 10 square meters, the latitude from the north pole is 0.747 radians, the relative dielectric constant of the reflecting surface is 80, the conductivity is 4.3 mhos per meter. The radius of the earth as calculated by the computer from Equation (190) is 6,366,878.23 meters. This yields an effective radius of 8,488,958.70 meters. Horizontal polarization is assumed.

Utilizing the data and a typical antenna pattern, Figure B-1 is sketched from values calculated by the computer logic previously listed. The computer traced out the noted lobes of Figure B-1 with ΔR_2 equal to 2,000 meters and ΔZ_2 equal to 2.5 meters. Z_1 equals 4,572 meters. Figure B-1 shows the coverage of the second, third, and fourth lobes obtained when a $4/3$ earth was assumed. The first lobe is omitted because it is very close to the surface of the earth. The two lobes which are larger were obtained using a geometric earth. The value assumed for η is hence seen to be very critical.

Table I and Table II list some of the major factors discussed in this paper calculated at various points on the lobes obtained from a $4/3$ earth consideration.

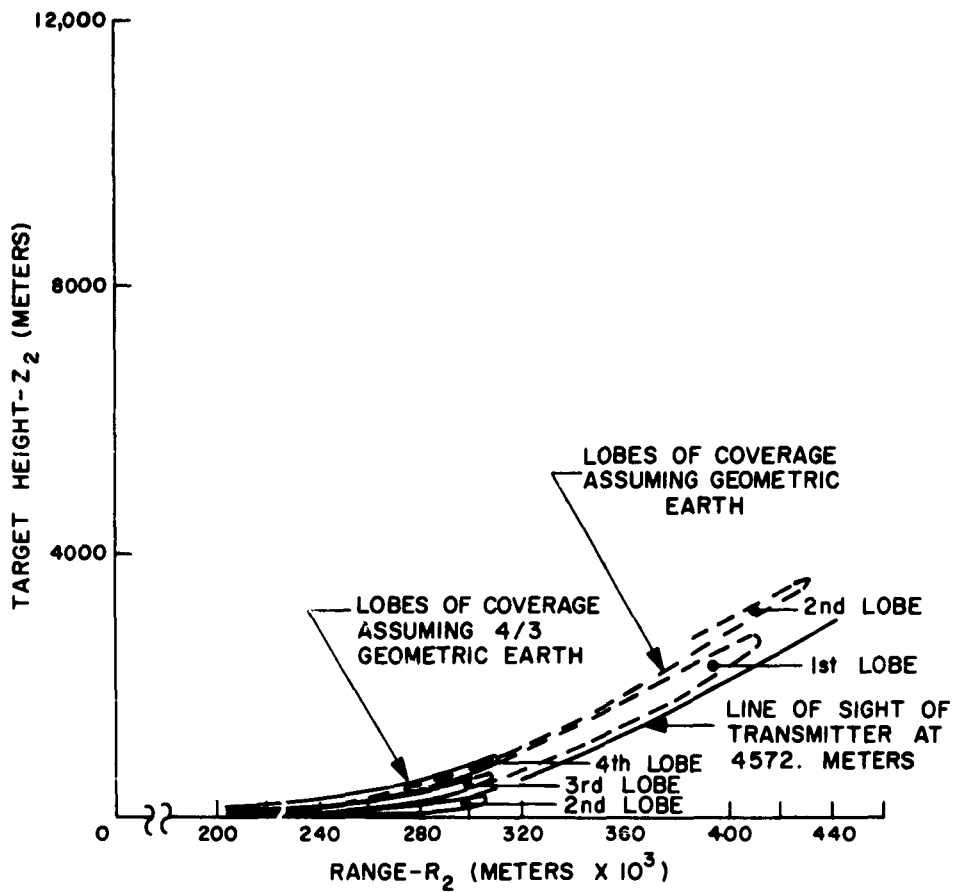


Figure B-1

Pattern of coverage of a radar with typical properties stated on page B-1. Geometrical and 4/3 earth coverage is noted.

Table I

Typical intermediate parameters necessary for the calculation of a coverage diagram.

$R_2 \times 10^3$	Z_2	$ K_2 F ^4$ Eq. (198)	$R_1 \times 10^3$ Eq. (121)	$R_3 \times 10^3$ Eq. (122)	ΔR Eq. (193)
226	55.5	.347	218	9.79	1.23
246	85.5	.584	233	14.2	.907
266	135	.504	242	23.0	.855
286	225	.944	248	39.4	.962
310	390	.980	251	58.2	1.057
286	262	.958	244	39.1	1.22
266	182	.442	239	26.8	1.26
246	122	.643	229	16.0	1.21
226	87.5	.256	216	9.28	1.23
206	62.5	.769	198	5.36	1.34
216	84.0	.303	208	9.25	1.63
236	119	.355	222	15.4	1.59
256	174	.344	233	24.6	1.56
276	264	.858	239	38.4	1.64
296	389	.938	243	54.0	1.69
296	418	.951	241	53.1	1.88
276	298	.965	236	38.6	1.91
256	213	.506	228	26.3	2.02
236	153	.217	217	17.2	2.13
216	113	.324	204	10.7	2.19
206	104	.281	198	8.91	2.23
226	141	.285	212	14.5	2.21

Table II

Typical intermediate parameters necessary for the calculation of a coverage diagram.

$R_2 \times 10^3$	Z_2	$\tan \alpha_1, \alpha_2$ Eq. (117) Eq. (118)	$\operatorname{Re}(\Gamma_h)$ Eq. (135)	D Eq. (145)	$\operatorname{Im}(\Gamma_h) \times 10^{-3}$ Eq. (135)
226	55.5	123	-.999	.916	+.616
246	85.5	170	-.999	.816	+.463
266	135	218	-.999	.687	+.388
286	225	264	-.999	.574	+.297
310	390	299	-.999	.478	+.257
286	262	241	-.999	.579	+.318
266	182	196	-.999	.685	+.391
246	122	157	-.999	.798	+.499
226	87.5	119	-.999	.890	+.654
206	62.5	91.4	-.998	.947	+.855
216	84.0	101	-.998	.916	+.759
236	119	132	-.999	.837	+.592
256	174	167	-.999	.735	+.470
276	264	199	-.999	.633	+.394
296	389	227	-.999	.550	+.344
296	418	218	-.999	.554	+.359
276	298	187	-.999	.634	+.412
256	213	156	-.999	.730	+.500
236	153	126	-.999	.825	+.621
216	113	98.4	-.998	.903	+.784
206	104	86.6	-.998	.923	+.900
226	141	109	-.998	.861	+.711